Mathematics (English Version)
Solutions for 2023 Exam

General Instructions

[The answers of the Question Nos 1, 2, 3, 4 are to be written at the beginning of the answer script mentioning the question numbers in the serial order. Necessary calculation and drawing must be given on the right hand side by drawing margins on the first few pages on the answer script. Tables and Calculators of any type are not allowed. Approximate value of π may be taken as $\frac{22}{7}$, if necessary: Graph paper will be supplied. If required Arithmetic problems may be solved by algebraic method.]

[Alternative Question No. 11 is given for sightless candidates on Page No. 15]
[Additional Question No. 16 is only for external candidates on Page No. 16]

1. Choose the correct answer from the following question: 1 x 6 = 6

(i) Three friends A, B, C started a business with Capitals Rs. x, 2x and y respectively, at the end of term profit is Rs. z, then the share of profit A is

(a) Rs. $\frac{xz}{3x+y}$

(b) Rs. $\frac{2xz}{3x+y}$

(c) Rs. $\frac{z}{2x+y}$

(d) Rs. $\frac{xyz}{3x+y}$

Answer: (a) Rs. $\frac{xz}{3x+y}$

Explanation: $Ratio\ of\ the\ share\ = x : 2x : y$

$Total\ amount\ = x + 2x + y = 3x + y$

$profit\ = z$

$A's\ profit\ share = \frac{A's\ share}{total\ investment} \times\ total\ profit$

A's profit = $\frac{x}{3x+y} \times z = \frac{xz}{3x+y}$

(ii) Number of solutions of the equation $x^2 = x$ are

(a) 1
(b) 2
(c) 0
(d) 3

Answer: (b) 2
Explanation:

$x^2 = x$
$x^2 - x = 0$
$x(x - 1) = 0$
$x = 0, x = 1$

(iii) If two circle touch each other internally, then the number of common tangents of the circles is

(a) 1
(b) 2
(c) 3
(d) 4

Answer: (b) 2
Explanation: If two circle touch each other internally, then the number of common tangents of the circles is 2.

(iv) For any value of 0 the maximum value of 5 + 4 $\sin sin\,\theta$ is

(a) 9
(b) 1
(c) 0
(d) 5

Answer: (a) 9
Explanation:
Maximum value of $\sin sin\,\theta = 1$ (at $\theta = 90°$)
Maximum value of 5 + 4 $\sin sin\,\theta$ = 5 + Max. value of $4 \sin sin\,\theta$

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Maximum value of 5 + 4 $\sin sin\,\theta$ = 5 + 4(1) = 9

(v) If the ratio of the volumes of two solid spheres is 27:8, then the ratio of their curved surface area is

(a) 1 : 2
(b) 9 : 4
(c) 1 : 8
(d) 1 : 16

Answer: (b) 9 : 4
Explanation:

$$Volume\ of\ Sphere\ 1 : Volume\ of\ Sphere\ 2\ =\ \frac{4}{3}\pi {r_1}^3 :\ \frac{4}{3}\pi {r_2}^3$$

$$27 : 8\ = \frac{4}{3}\pi {r_1}^3 :\ \frac{4}{3}\pi {r_2}^3$$

$$27 : 8\ = {r_1}^3 :\ {r_2}^3$$

$$\frac{27}{8} =\ \frac{{r_1}^3}{{r_2}^3}$$

$$\frac{r_1}{r_2} = \frac{3}{2}$$

$$r_1\ = \frac{3}{2} r_2$$

$$Surface\ area\ of\ Sphere\ 1 : Surface\ area\ of\ Sphere\ 2\ =\ 4\pi {r_1}^2 : 4\pi {r_2}^2$$

$$= \frac{{r_1}^2}{{r_2}^2}$$

$$= \frac{{(\frac{3}{2} r_2)}^2}{{r_2}^2}$$

$$= \frac{9}{4}$$

$$Surface\ area\ of\ Sphere\ 1 : Surface\ area\ of\ Sphere\ 2\ =\ 9 : 4$$

(vi) Three values of a variable are 4, 5 and 7, if their frequencies are p – 2, p + 1, and p – 1 respectively and Mean is 5.4, then value of p is:

(a) 1
(b) 2
(c) 3
(d) 4

Answer: (d) 4
Explanation:

$$A \cdot M = \frac{\Sigma fx}{\Sigma f} = \frac{4(P-2)+5(P+1)+7(P-1)}{P-2+P+x+P-x}$$
$$= \frac{4P-8+5P+5+7P-7}{3P-2}$$
$$= \frac{6P-10}{3P-2}$$
$$\therefore \frac{16P-10}{3P-2} = 5.4$$
$$16P - 10 = 5.4\,(3P - 2)$$
$$16P - 10 = 16.2P - 10.8$$
$$10.8 - 10 = 16.2\,P - 16\,P$$
$$0.8 = 0.2\,P$$
$$P = 4$$

2. Fill up the blanks (any **five**): 1 x 5 = 5

(i) If the amount of Rs. 180 after one year be Rs. 198, then the rate of Simple Interest is ____.

Answer: 10 %
Explanation:

$$Amount = Principal + Simple\ Interest$$
$$198 = 180 + Simple\ Interest$$
$$Simple\ Interest = 198 - 180 = Rs.\ 18$$
$$Simple\ Interest = \frac{PNR}{100}$$
$$18 = \frac{180 \times 1 \times R}{100}$$
$$R = \frac{18 \times 100}{180 \times 1}$$
$$R = 10\ \%$$

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(ii) If mean proportional of (a^2bc) and $(4bc)$ is *x*, then the value of *x* is ______.

Answer: $\pm 2abc$
Explanation:

$$\frac{a^2bc}{x} = \frac{x}{4bc}$$
$$4a^2b^2c^2 = x^2$$
$$x = \sqrt{4a^2b^2c^2}$$
$$x = \pm 2abc$$

(iii) If $tan\theta cos60^{\circ} = \frac{\sqrt{3}}{2}$, then the value of $sin\left(\theta - 15^{\circ}\right)$ is _____.

Answer: $\frac{1}{2}$
Explanation:
Given that, $an\theta cos60^{\circ} = \frac{\sqrt{3}}{2}$ **,** where the value of $cos60^{\circ} = \frac{\sqrt{3}}{2}$

$$\tan tan\, \theta\, .\, \left(\frac{\sqrt{3}}{2}\right) = \left(\frac{\sqrt{3}}{2}\right)$$
$$\tan tan\, \theta\, = 1$$
$$\therefore \theta\, = 45^{\circ}$$

Now substitute the value of θ in $sin\left(\theta - 15^{\circ}\right)$**,**

$$\sin sin\left(45^{\circ} - 15^{\circ}\right) = \sin sin\ 30^{\circ} = \frac{1}{2}$$

(iv) If $\angle A$ and $\angle B$ are complementary then $\angle A + \angle B =$ _____.

Answer: 90°

Explanation: If $\angle A$ and $\angle B$ are complementary then $\angle A + \angle B = 90^{\circ}$

(v) The median of the numbers $8, 15, 10, 11, 7, 9, 11, 13$ and 16 is _____.

Answer: 11

Explanation:

The value of the middle-most observation is called the median of the data.
First arrange the given numbers in ascending order,
$7,\ 8,\ 9,\ 10,\ 11,\ 11,\ 13,\ 15,\ 16$
Here $n = 9$, which is odd.
Where, n is the number of the given number.

$\therefore median = value\ of\ \frac{1}{2}(n+1)^{th}\ observation.$
$= \frac{1}{2}(9+1)$
$= \frac{1}{2}(10)$
$= \frac{10}{2}$
$= 5$
Then, value of 5^{th} term $= 11$
Hence, the median is 11.

(vi) The shape of a pencil with one end sharpened is the combination of a ____and a _____.

Answer: Cone and Cylinder
Explanation: The shape of a pencil with one end sharpened is the combination of a cone and a cylinder.

3. Write **True** or **False** (any **five**): 1 x 5 = 5
(i) In compound interest if the rate of interest in first three years is $r_1\%,\ r_2\%,\ 2r_3\%$ respectively, then the amount for the principal P at the end of three years is $P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right)$.

Answer: False

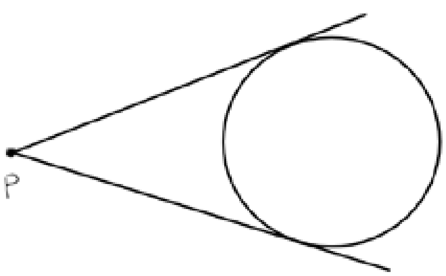
Explanation: In compound interest if the rate of interest in first three years is $r_1\%,\ r_2\%,\ 2r_3\%$ respectively, then the amount for the principal P at the end of three years is $P\left(1 + \frac{r_1}{100}\right)\left(1 + \frac{r_2}{100}\right)\left(1 + \frac{r_3}{100}\right)$.

(ii) The values of $cos36^\circ$ and $sin54^\circ$ are equal.

Answer: True
Explanation: Since, $36° + 54° = 90°$
Thus $cos36^\circ = cos(90 - 54) = sin54^\circ$
Thus it is True

(iii) One tangent can be drawn on a circle from an external point.

Answer: False
Explanation:
This statement is false. As maximum of two tangents can be drawn on a circle from an external point.



(iv) The compound ratio of $2ab: c^2, bc: a^2$ and $ca: 2b^2$ is $1: 1$.

Answer: True

Explanation:

$ab : c^2, bc : a^2, ca : b^2$

= $(ab \times bc \times ca) : \left(c^2 \times a^2 \times b^2\right)$

= $\frac{ab \times bc \times ca}{c^2 \times a^2 \times b^2}$

= $\frac{a^2 \times b^2 \times c^2}{a^2 \times b^2 \times c^2} = \frac{1}{1} = 1 : 1 \ (Proved)$

(v) If the numerical values of curved surface area and volume of a sphere are equal, then radius will be 3 units.

Answer: True

Explanation:

$C.\ S.\ A\ of\ sphere\ =\ Volume\ of\ sphere$

$4\pi r^2 = \frac{4}{3}\pi r^3$

$1 = \frac{1}{3}r$

$r = 3\ units$

Hence, it is True.

(vi) The Mode of the data $5, 2, 4, 3, 5, 2, 5, 2, 5, 2$ is 2.

Answer: False

Explanation:

This is a false statement. As the mode of the data is the observation having maximum occurrence.

Observation	Frequency
2	4

3	1
4	1
5	4

Thus the mode is 2 as well as 5.

4. Answer the following questions (any **ten**): 2 x 10 = 20

(i) Find the rate of simple interest per annum when the interest of some money in 5 years will be $\frac{2}{5}$ part of its principal.

Answer: 8%

Explanation: S.I. $= (2/5)$ of Principal

Time $=$ years

Formula used:

Amount $= P + \frac{P \times R \times T}{100}$

Where P, R and T represents Principal, rate and time respectively.

Calculation:

$$S.I. = \frac{P \times R \times T}{100}$$

$$\Rightarrow \left(\frac{2}{5}\right) of\ P = \frac{P \times R \times 5}{100}$$

$$\Rightarrow \left(\frac{2}{5}\right) = \frac{R \times 5}{100}$$

$$\Rightarrow \left(\frac{2}{5}\right) = \frac{R}{20}$$

$$\Rightarrow R = 8\%$$

∴ Rate is 8% per annum.

(ii) In a business capitals of A and B are in the ratio $\frac{1}{7}:\frac{1}{4}$. If they make a profit of Rs. 11,000 at the end of the year, calculate the share of their profit.

Answer: A’s profit = Rs $4,000$, B’s profit = Rs 7000

Explanation:

$A:B = \frac{1}{7}:\frac{1}{4}$

We can write it as

$A:B = \frac{4}{28}:\frac{7}{28}$

Now $A + B = \frac{11}{28}$

Division of money:

Now whole money can be considered as:

$\Rightarrow 11000\times\frac{28}{11} \quad \Rightarrow 28,000$

A's part will be -

$\Rightarrow 28,000\times\frac{1}{7} \quad \Rightarrow 4,000.$

B’s part will be -

$\Rightarrow 28000\times\frac{1}{4} \quad \Rightarrow 7000.$

(iii) If the sum of the roots of the equation $x^2 - x = k(2x - 1)$ is zero, find k.

Answer: $k = \frac{-1}{2}$

Explanation:

$x^2 - x = 2kx - k$

$x^2 - (2k + 1)x + K = 0$

$sum\ of\ roots = 2K + 1 = 0$

$k = \frac{-1}{2}$

(iv) If $b \propto a^3$ and a increases in the ratio of $2:3$, let us find in what ratio b will be increased.

Answer: $8:27$

Explanation:

Given: $b \propto a^3$

$$b = ka^3$$

$$\frac{b_1}{b_2} = \frac{k}{k}\frac{a_1^{\ 3}}{a_2^{\ 3}}$$

$$\frac{b_1}{b_2} = \frac{(2)^3}{(3)^3}$$

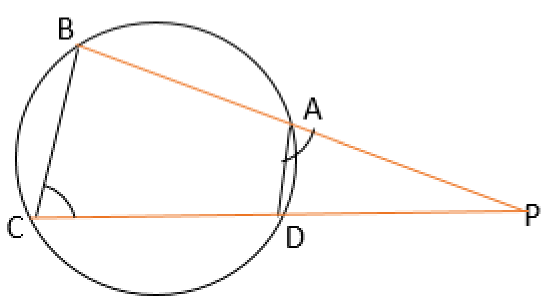
$$\frac{b_1}{b_2} = \frac{8}{27}$$

Hence, b must increase in ratio $8:27$.

(v) AB and CD are two chords of a circle. If we extend BA and DC, they intersect each other at the point P. Prove that $\angle PCB = \angle PAD$.

Explanation:

Given, AB and CD are two chords of a circle



ABCD is a cyclic quadrilateral,

So, the opposite angles in a cyclic quadrilateral **is equal to 180°**

$$\Rightarrow \angle BCD + \angle BAD = 180°$$
$$\Rightarrow \angle BAD = 180° - \angle BCD$$

$\Rightarrow\angle BAD = 180° - \angle PCB$...$(i)$ (Since, $\angle BCD = \angle PCB$)

From the fig, $\Rightarrow\angle PAD + \angle BAD = 180°$ (Linear pair)

$\Rightarrow\angle PAD = 180° - \angle BAD$

From eq (i), $\Rightarrow\angle PAD = 180° - (180° - \angle PCB)$

$\Rightarrow\angle PAD = 180° - 180° + \angle PCB$

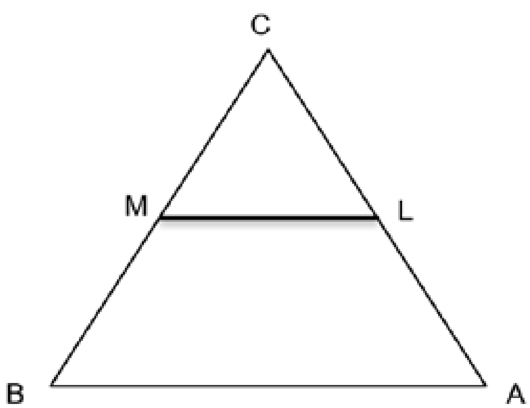
$\Rightarrow\angle PAD = \angle PCB$ (proved)

(vi) In $\triangle ABC, L$ and M are two points on the sides AC and BC respectively such that $LM \parallel AB$ and $AL = (x - 2)$ unit, $AC = 2x + 3$ unit, $BM = (x - 3)$ unit and $BC = 2x$ unit. Determine the value of x.

Answer: $x = 9$

Explanation:

Given



$AL = x - 2$

$AC = 2x + 3$

$BM = x - 3$

$BC = 2x$

$\Delta MCL \sim \Delta BCA$

$\frac{AL}{AC} = \frac{BM}{BC}$

$\frac{x-2}{2x+3} = \frac{x-3}{2x}$

$2x^2 - 4x = 2x^2 - 6x + 3x - 9$

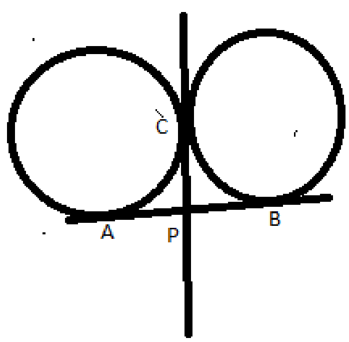
$-4x + 6x - 3x = -9$

$x = 9$

(vii) Two circles touch each other externally at the point C. A direct common tangent AB touches the two circles at the points A and B. Find the value of ∠ACB.

Answer: 90°

Explanation:



Given X and Y are two circles touch each other externally at C. AB is the common tangent to the circles X and Y at point A and B respectively.

To find : $\angle ACB$

Proof:

Let P be a point on AB such that, PC is at right angles to the Line Joining the centers of the circles.

Note that, PC is a common tangent to both circles.

This is because tangent is perpendicular to radius at point of contact for any circle.

let $\angle PAC = \alpha$ and $\angle PBC = \beta$.

$PA = PC$ [lengths of the tangents from an external point C]

In a triangle CAP, $\angle PAC = \angle ACP = \alpha$

similarly $PB = CP$ and $\angle PCB = \angle CBP = \beta$

now in the triangle ACB

$\angle CAB + \angle CBA + \angle ACB = 180^{\circ}$ [sum of the interior angles in a triangle]

$\alpha + \beta + (\alpha + \beta) = 180^{\circ}$ (Since $\angle ACB = \angle ACP + \angle PCB = \alpha + \beta$.

$2\alpha + 2\beta = 180^{\circ} \alpha + \beta = 90^{\circ} \therefore \angle ACB = \alpha + \beta = 90^{\circ}$

(viii) If $tan2\,A = cot\left(A - 30^{\circ}\right)$, then find the value of $sec\left(A + 20^{\circ}\right)$

Answer: 2

Explanation: Given:

$tan2A = cot\left(A - 30^{\circ}\right)$,
Formula Used:
If $tanA = tanB$ then $A = B$
If $cotA = cotB$ then $A = B$
Calculation:

We have $tan2A = cot\left(A - 30^{\circ}\right)$

$\Rightarrow cot\left(90^{\circ} - 2A\right) = cot\left(A - 30^{\circ}\right)$,

$\Rightarrow \left(90^{\circ} - 2A\right) = \left(A - 30^{\circ}\right)$,

$\Rightarrow A = 40^{\circ}$

$\therefore$ Sec(A+20°) = $40^{\circ} + 20^{\circ}$

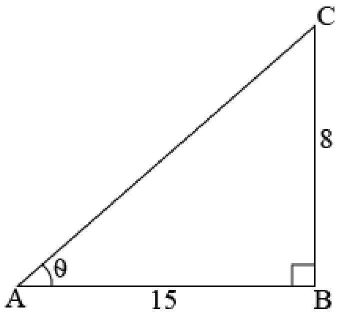
Sec(60°) = 2

(ix) If $tan\theta = \frac{8}{15}$, find the value of $sin\theta$

Answer: $\frac{8}{17}$

Explanation: Given, θ is an acute angle and $tan\theta = \frac{8}{15}$.

In figure we have,



$tan\theta = \frac{BC}{AB} = \frac{8}{15}$

So, $BC = 8$ and $AB = 15$

By Pythagoras theorem, we have

$$AC = \sqrt{\left(AB^2 + BC^2\right)} = \sqrt{\left(5^2 + 8^2\right)} = \sqrt{(25 + 64)} = \sqrt{289} \Rightarrow AC = 17$$

Now, Sin θ = $\frac{BC}{AC} = \frac{8}{17}$

(x) If the volume of a right circular cone is V cubic unit, base area is A sq. unit and height is H unit, then find the value of $\frac{AH}{3V}$,

Answer: 1

Explanation: Let the radius of the right circular cone is r unit. ∴ the volume of the cone, $V = \frac{1}{3}\pi r^2 H$ cubic - unit

Again , the area of the base of the cone, $A = \pi r^2$ sq-unit.

$$\therefore \frac{AH}{V} = \frac{\pi r^2 H}{\frac{1}{3}\pi r^2 H} \Rightarrow \frac{AH}{V} = 3$$

Hence the value of $\frac{AH}{3V} = 1$.

(xi) Find the ratio of the volumes of a solid right circular cylinder and a solid right circular cone of equal radii and equal heights.

Answer: $3:1$

Explanation: Volume of circular cylinder $= \pi r^2 h$

Volume of cone $= \frac{1}{3}\pi r^2 h$

$\therefore$ Their Ratio $= \frac{\pi r^2 h}{\frac{1}{3}\pi r^2 h} = \frac{3}{1} = 3:1$

(xii) If $6, 8, 10, 12, 13, x$ are in increasing order and their mean and median are equal, then find the value of x.

Answer: $x = 17$

Explanation: Numbers in ascending order are $6, 8, 10, 12, 13, x$.

Mean $= \frac{6+8+10+12+13+x}{6} = \frac{49+x}{6}$

No. of terms (n) $= 6$ (even)

$$Median = \frac{\left(\frac{n}{2}\right)^{th} term + \left(\frac{n}{2}+1\right)^{th} term}{2} \quad Median = \frac{\left(\frac{6}{2}\right)^{th} term + \left(\frac{6}{2}+1\right)^{th} term}{2} = \frac{3^{rd} + 4^{th}}{2} = \frac{10+12}{2} = \frac{22}{2} =$$

According to given condition

$$\frac{49+x}{6} = 11 \Rightarrow 49 + x = 66$$

$$\Rightarrow x = 17$$

5. Answer any **one** question: 5

(i) The number of smokers is decreasing at the rate $6\frac{1}{4}\%$ per year due to publicity of anti-smoking. If at present number of smokers in a town are 22500, find the number of smokers of that town 2 years ago.

Answer: 25600

Explanation:

$$A = 22500, \quad P = ? \; r = 6\frac{1}{4}\% = \frac{25}{4}\%, \; n = 2$$

$$A = P\left(1 - \frac{r}{100}\right)^n$$

$$22500 = P\left(1 - \frac{25}{4\times100}\right)^2$$
$$22500 = P\left(1 - \frac{1}{16}\right)^2$$
$$22500 = P \times \left(\frac{15}{16}\right)^2$$
$$\frac{22500\times16\times16}{15\times15} = P$$
$$P = 25600$$

Thus, 2 years ago there were 25600 smokers.

(ii) In a partnership business, the ratio of capitals of three friends is $6:4:3$. After 4 months 1^{st} friend withdraws his half of the capital and after 8 more months total profit is Rs. 61,050 . Find the share of profit of three friends.

Answer: $Profit\ of\ A = Rs.\ 22200$, Profit of B = Rs. 22200, Profit of C = Rs. 16650

Explanation:

Let the capital of $A = 6x$

Let the capital of $B = 4x$

Let the capital of $C = 3x$

Half of $6x = \frac{6x}{2} = 3x$

Ratio of capital with respect of 1 month.

$$= [(6x\times4) + (3x\times8)]: (4x\times12): (3x\times12)$$
$$= (24x + 24x): 48x : 36x$$
$$= 48x : 48x : 36x$$
$$= 4: 4: 3$$
$$Total\ profit = ₹\,61,050$$
$$Profit\ of\ A = Share\ of\ A \times total\ profit$$
$$= \frac{4}{4+4+3}\times61050$$

$= \frac{4}{11} \times 61050$
$= 4 \times 5550$
$= Rs.\ 22200$
$Profit\ of\ A = Rs.\ 22200$
Similarly, Profit of B = Rs. 22200 (since the ratio of B is same as A)

Profit of C = 3×5550 = Rs. 16650

6. Answer any **one** question: 3

(i) Solve: $\frac{x-3}{x+3} - \frac{x+3}{x-3} + 6\frac{6}{7} = 0, (x \neq 3, -3)$

Answer: $x = 4, \frac{-9}{4}$

Explanation:

$\frac{x^2-6x+9-(x^2+6x+9)}{x^2-9} + \frac{48}{7} = 0$

$\frac{x^2-6x+5-5y^2-6x-y}{x^2-9} + \frac{48}{7} = 0$

$\frac{-12x}{x^2-9} + \frac{48}{7} = 0$

$-84x + 48x^2 - 432 = 0$

$-7x + 4x^2 - 36 = 0$

$4x^2 - 7x - 36 = 0$

$4x^2 - (16x - 9x) - 36 = 0$

$4x^2 - 16x + 9x - 36 = 0$

$4x(x-4) + 9(x-4) = 0$

$(x-4)(4x+9) = 0$

$x = 4, \frac{-9}{4}$

(ii). If the price of 1 dozen pens is reduced by Rs. 6 , then 3 more pens will be got in Rs. 30. Calculate the price of 1 dozen pens before the reduction of price.

Answer: Rs 30

Explanation:

Let the price of 1 dozen pen at present is $Rs\ x$.

$\therefore$ In Rs at present $\frac{12}{x}\times 30$ pen $= \frac{360}{x}$ pen are got.

If the price is reduced by RS 6 per dozen, then it becomes$RS(x - 6)$.

Then for Rs 30 we get, $\frac{12}{x-6}\times 30$ pens $= \frac{360}{x-6}$ pens.

As per question, $\frac{360}{x-6} - \frac{360}{x} = 3$.

or,

$360\left(\frac{1}{x-6} - \frac{1}{x}\right) = 3\ or\ , 120\left\{\frac{x-x+6}{(x-6)x}\right\} = 1$

or, $120\left(\frac{6}{x^2-6x}\right) = 1$

or, $\frac{720}{x^2-6x} = 1$ or,$x^2 - 6x = 720$

or, $x^2 - 6x - 720 = 0$

or, $x^2 - (30 - 24)x - 720 = 0$

or, $x^2 - 30x + 24x - 720 = 0$

or, $x(x - 30) + 24(x - 30) = 0$

or, $(x - 30)(x + 24) = 0$

$\therefore$ either $x - 30 = 0$ or,$x + 20 = 0$

$\Rightarrow x = 30$ or,$x =- 24$.

But price of pens can not be negative, $\therefore x \neq - 24$, $\therefore x = 30$

Hence, before reduction of prices, the price of 1 dozen pen was Rs 30.

7. Answer any **one** question: 3

(i) If $x = \frac{1}{2-\sqrt{3}}$ and $y = \frac{1}{2+\sqrt{3}}$, then find the value of $\frac{1}{x+1} + \frac{1}{y+1}$.

Answer: 1

Explanation:

$x = \frac{1}{2-\sqrt{3}} \Rightarrow x + 1 = \frac{1}{2-\sqrt{3}} + \frac{1}{1}$

$= \frac{1+2-\sqrt{3}}{2-\sqrt{3}}$

$= \frac{3-\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{6+3\sqrt{3}-2\sqrt{3}-3}{4-3}$

$= 3 + \sqrt{3}$

$y = \frac{1}{2+\sqrt{3}} \Rightarrow y + 1 = \frac{1}{2+\sqrt{3}} + 1$

$= \frac{1+2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{3+\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= 6 - 3\sqrt{3} + 2\sqrt{3} - 3$

$= 3 - \sqrt{3}$

$\Rightarrow \frac{1}{x+1} + \frac{1}{y+1} \Rightarrow \frac{1}{3+\sqrt{3}} + \frac{1}{3-\sqrt{3}} = \frac{3-\sqrt{3}+3+\sqrt{3}}{9-3} = \frac{6}{6} = 1$

(ii) If $x \propto y$ and $y \propto z$, then show that $\frac{x}{yz} + \frac{y}{zx} + \frac{z}{xy} \propto \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$.

Explanation:

$Given$: $x \,\alpha\, y \Rightarrow x = k_1 y$

$y \,\alpha\, z \Rightarrow y = k_2 z$ ....(i)

$\Rightarrow x = k_1\left(k_2 z\right) \Rightarrow x = k_1 k_2 z$ ...(ii)

The given expression is –

$\frac{x}{yz} + \frac{y}{yx} + \frac{z}{xy} \propto \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$

$\frac{x^2+y^2+z^2}{xyz} \propto \frac{yz+xz+xy}{xyz}$

$\frac{x^2+y^2+z^2}{xy+yz+zx} = k$

To prove the proportionality for the given expression, the value of $\frac{x^2+y^2+z^2}{xy+yz+zx}$ should be non-zero constant.

Now substitute the values of x and y from (i) and (ii),

$$\frac{k_1^2k_2^2z^2+k^2z^2+z^2}{k_1k_2z\left(k_2z\right)+k_2z(z)+z\left(k_1k_2z\right)}$$
$$=\frac{\left(k_1^2k_2^2+k_2^2+1\right)z^2}{\left(k_1k_2^2+k_2+k_1k_2\right)z^2}$$
$$=\frac{k_1^2k_2^2+k_2^2+1}{k_1k_2^2+k_2+k_1k_2}=\ Non\ -\ zero\ constant$$

[$Since\ k_1$ & k_2 are non-zero separation constant.]

$\therefore \frac{x}{yz}+\frac{y}{zx}+\frac{z}{xy}\propto\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$

8. Answer any **one** question: 3

(i) If $\frac{a^2}{b+c}=\frac{b^2}{c+a}=\frac{c^2}{a+b}=1$, then show that $\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}=1$.

Explanation:

$\frac{a^2}{b+c}=1$ or $\frac{a}{b+c}=\frac{1}{a}$ [by componendo process]

$\therefore\frac{1}{1+a}=\frac{a}{a+b+c}$……………..(1)

Similarly , $\frac{1}{1+b}=\frac{b}{a+b+c}$………(2)

and , $\frac{1}{1+c}=\frac{c}{a+b+c}$…………….(3)

Now, adding (1), (2) and (3) we get,

$$\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}=\frac{a}{a+b+c}+\frac{b}{a+b+c}+\frac{c}{a+b+c}$$
$$=\frac{a+b+c}{a+b+c}=1$$
$$\therefore\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}=1$$

(ii) If the fourth and fifth of the five numbers in continued proportion are 54 and 162 respectively, find the first number.

Answer: 2

Explanation:

Let the continued proportion be a, ak^2ak^2, ak^3, ak^4

Given, $ak^3 = 54$ and $ak^4 = 162$

Now, $ak^4 = 162$

$\Rightarrow ak^3(k) = 162$
$\Rightarrow 54(k) = 162$
$\Rightarrow k = 3$

Given $\Rightarrow ak^3 = 54$

$a(3)^3 = 54$
$a = 2$

So, first number will be '2'

9. Answer any **one** question: 5

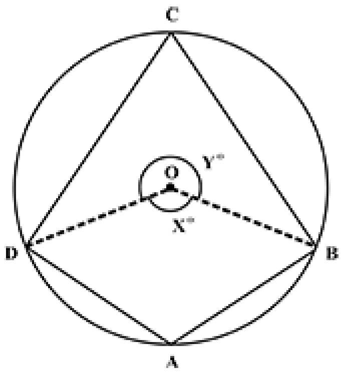
(i) $Prove\ that\ in\ a\ cyclic\ quadrilateral\ opposite\ angles\ are\ supplementary.$

Explaination:

Given: A cyclic quadrilateral $ABCD$.

To Prove: $\angle A + \angle C = 180^\circ$ and $\angle B + \angle D = 180^\circ$

Construction: Let O be the centre of the circle. Join O to B and D. Then let the angle subtended by the minor arc and the major arc at the centre be x° and y° respectively.



Proof: $x^{\circ} = 2\angle C$ [Angle at centre theorem] -----------------(i)

$y^{\circ} = 2\angle A$ -----------------(ii)

Adding (i) and (ii), we get

$x^{\circ} + y^{\circ} = 2\angle C + 2\angle A$ -----------------(iii)

But, $x^{\circ} + y^{\circ} = 360^{\circ}$ -----------------(iv)

From (iii) and (iv), we get,

$2\angle C + 2\angle A = 360^{\circ} \Rightarrow \angle C + \angle A = 180^{\circ}$

But we know that angle sum property of quadrilateral,

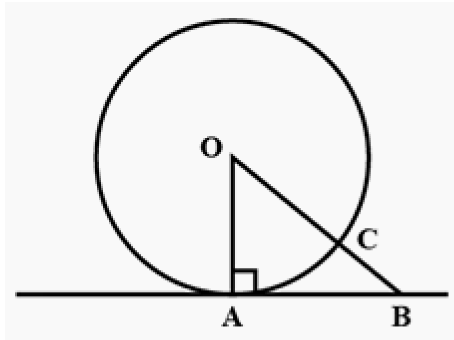
$\angle A + \angle B + \angle C + \angle D = 360^{\circ}$ $\angle B + \angle D + 180^{\circ} = 360^{\circ}$ $\angle B + \angle D = 180^{\circ}$

Hence proved.

(ii) Prove that the tangent to a circle at any point on it is perpendicular to the radius that passes through the point of contact.

Explanation:

Referring to the figure:



$OA = OC$ (Radii of circle)
Now $OB = OC + BC$
$\therefore OB > OC$ (OC being radius and B any point on tangent)

$\Rightarrow OA < OB$
B is an arbitrary point on the tangent.

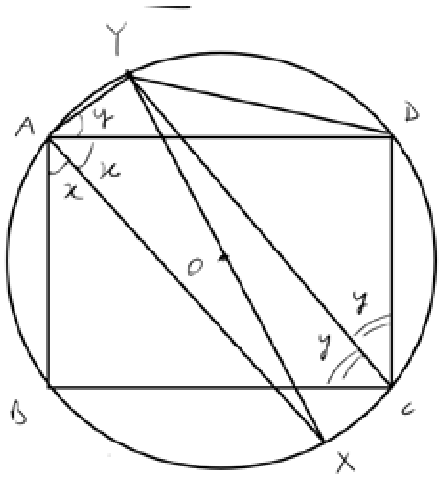
Thus, OA is shorter than any other line segment joining O to any point on tangent.
Shortest distance of a point from a given line is the perpendicular distance from that line.
Hence, the tangent at any point of circle is perpendicular to the radius.

10. Answer any **one** question: 3

(i) $ABCD$ is a cyclic quadrilateral. Bisectors of $\angle DAB$ and $\angle BCD$ intersect the circle at X and Y respectively. If O be the centre of the circle, find $\angle XOY$.

Answer: 180°

Explanation:



$\angle A + \angle B = 180^{\circ}$ (opposite angles of cyclic quadrilateral)

$\frac{\angle A}{2} + \frac{\angle B}{2} = 90^{\circ}$

Or, $\angle DAX + \angle DCA = 90^{\circ}$

Let $\angle DAX = x$ and $\angle DCA = y$

$x + y = 90^{\circ}$

Draw DY and AY

$\angle YAD = \angle DCY = y$ (Angles on same cord are equal)

Thus,

$or,\ \angle YAX = 90^{\circ}$

$\angle YAD + \angle DAX = x + y = 90^{\circ}$

$Now,\ \angle XOY = 2\times\angle YAX$ ($the\ angle\ at\ the\ centre\ is\ double\ the\ angle\ at\ the\ circumference.$)

$Thus,\ \angle XOY = 2\times 90^{\circ} = 180^{\circ}$

(ii) Prove that a cyclic trapezium is an isosceles trapezium.

Solution:

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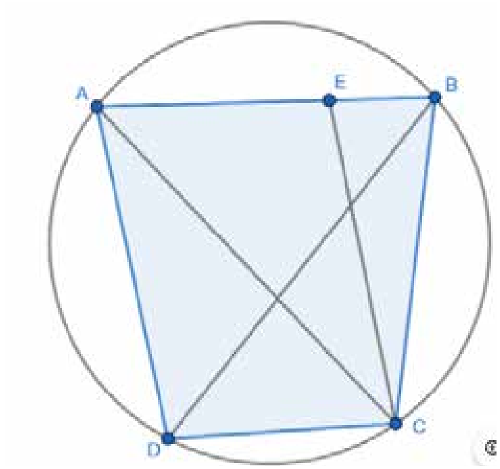

AC and BD are the diagonals and the line CE is parallel to AD. We know that the opposite angles of a parallelogram are equal.

$$\angle D = \angle AEC \ldots \ldots\ldots (1)$$

ow we also know that the sum of opposite angles of a cyclic quadrilateral is 180 degree.

$$\angle D + \angle ABC = 180 \ldots\ldots (2)$$

From (1) and (2) we can say that,

$$\angle AEC + \angle ABC = 180 \ldots\ldots (3)$$

Now angle AEC and CEB are linear pair,

$$\angle AEC + \angle CEB = 180 \ldots\ldots (4)$$

Now again from (3) and (4) we can say that,

$$\angle ABC = \angle CEB$$

We know that the sides opposite the equal angles are also equal.

$$\angle ABC = \angle CEB$$
$$CE = CB \ldots (5)$$

But we know that $AECD$ is a parallelogram and hence opposite sides must be equal.

$$\therefore CE = AD \ldots (6)$$

Therefore, from (5) and (6) we can say that

$$AD = CB \ldots .\ (7)$$

Thus the cyclic trapezium $ABCD$ is isosceles.

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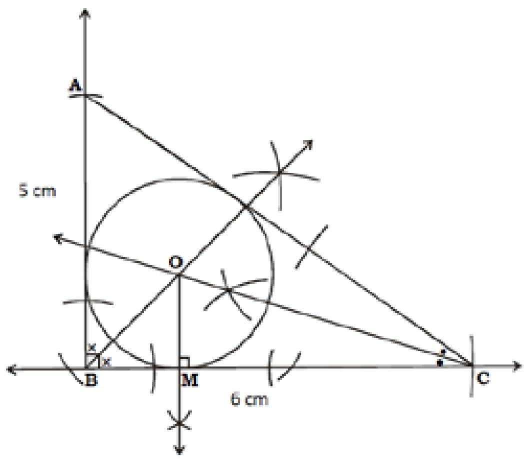
11. Answer any **one** question: 5

(i) Draw a right angled triangle of which two sides containing the right angle have the lengths $5\ cm$ and $6\ cm$. Now draw an incircle of the triangle.

Explanation:



Steps of construction:

1. Draw a line BC of 6 cm.
2. At point B draw a right angle.
3. Take a distance of 5 cm and cut an arc from the point B. This will give the point A.
4. Join A to C. This is the right angle triangle ABC.
5. Draw angle bisector of any two angles say ∠B and ∠C of △ABC and let these intersect at a point say O.
6. Taking O as centre and OM as radius, draw a circle.
7. The circle touches the other two sides of triangle. This will be the required in circle of the triangle.

(ii) Construct a square of equal area of an equilateral triangle of side 7 cm.

Explanation:

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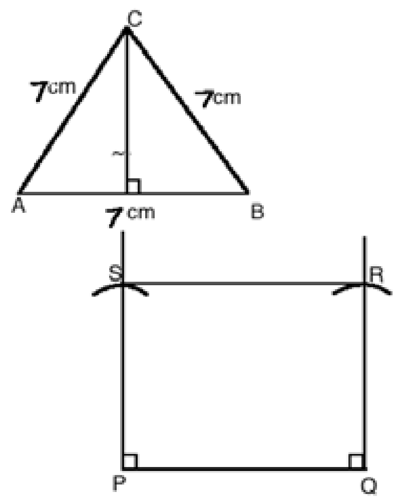
Area of equilateral triangle = $\frac{\sqrt{3}}{4}(side)^2$=$\frac{\sqrt{3}}{4}(7)^2$= 21.19

Solution: Steps of construction:

1 Draw equilateral $\triangle ABC$ according to the given measures.



2 Draw the perpendicular CD from C to AB.

Now CD is the height $h = \sqrt{7^2 - \left(\frac{7}{2}\right)^2} = \sqrt{\frac{147}{4}} = 6.06\ cm$ of the triangle. Draw perpendicular bisector of CD to get $DE = h/2$.

$$Area\ of\ \triangle ABC = \frac{1}{2}\times AB\times h = AB\times\frac{h}{2}$$

To draw a square of area $AB\times(h/2)$, the side of the square should be

$$\sqrt{AB\times\frac{h}{2}} = \sqrt{7\times\frac{6.06}{2}} = 4.6\ cm$$

Steps of construction:

1 Draw $Q = 4.6\ cm$.

2 At the point Q and P, draw angles of 90° on the same side of PQ.

3 Cut those perpendicular lines at $4.6\ cm$ at R and S.

4 Join RS.

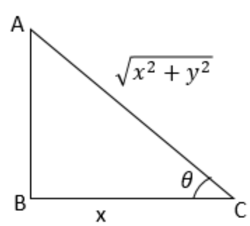
5 Area of square $PQ \times QR = 4.6 \times 4.6\ cm^2 \approx 21.16\ cm^2$

12. Answer any **two** questions: 3 x 2 = 6

(i). If $cos\theta = \frac{x}{\sqrt{x^2+y^2}}$, then prove that $x \sin sin\ \theta = y\ cos\theta$.

Explanation:

Given, $\cos cos\ \theta = \frac{x}{\sqrt{x^2+y^2}} = \frac{Adjacent\ side}{Hyp}$



By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$
$\Rightarrow (\sqrt{x^2+y^2})^2 = AB^2 + x^2$
$\Rightarrow x^2 + y^2 = x^2 + AB^2$
$\Rightarrow y^2 = AB$
$\Rightarrow AB = y$
We know that, $\theta = \frac{Opposite\ side}{Hyp} = \frac{y}{\sqrt{x^2+y^2}}$

LHS= $x \sin sin\ \theta = x \times \frac{y}{\sqrt{x^2+y^2}} = \frac{xy}{\sqrt{x^2+y^2}}$

RHS= $y\ cos\theta\ =\ y\times\frac{x}{\sqrt{x^2+y^2}}\ =\ \frac{xy}{\sqrt{x^2+y^2}}$

$\therefore LHS\ =\ RHS$
Hence, it is proved.

(ii) Radius of a circle is $7\ cm$. Find the angle in radians which subtains by an arc of this circle of length $5.5\ cm$ at the centre of the circle.

Answer: 45°

Explanation:

Length of $arc\ =\ 5.5\ cm$ and Radius of the circle $=\ 7\ cm$
Angle subtended by arc at the centre $=\ \frac{Length\ of\ arc}{Radius\ of\ the\ circle}$
⇒ Angle subtended by arc at the centre $=\ \frac{5.5}{7}\ =\ \frac{11}{14}$ radians
Since 1 radian $=\ \left(\frac{180}{\pi}\right)$
∴ Anqle subtended by arc at the centre $=\ \frac{11}{14}\ \times\ \left(\frac{180}{\pi}\right)\ =\ \frac{11}{14}\times180\times\ \frac{7}{22}\ =\ 45^{\circ}$ or $\frac{\pi}{4}$.

(iii) Show that $\frac{tan\theta+sec\theta-1}{tan\theta-sec\theta+1}\ =\ \frac{1+sin\theta}{cos\theta}$.

Explanation:

$LHS\ =\ \frac{tan\theta+sec\theta-1}{tan\theta-sec\theta+1}$

$=\ \frac{\frac{sinsin\ \theta}{coscos\ \theta}+\frac{1}{coscos\ \theta}-1}{\frac{sinsin\ \theta}{coscos\ \theta}-\frac{1}{coscos\ \theta}+1}$

$=\frac{sin\theta+1-cos\theta}{(sin\theta-1)+cos\theta}\times\frac{(sin\theta-1)-cos\theta}{sin\theta-1-cos\theta}$

$=\ \frac{(sin\theta-cos\theta)+1}{sin\theta+cos\theta-1}\times\frac{sin\theta+cos\theta)+1}{sin\theta+cos\theta+1}$

$=\ \frac{\theta\ +sinsin\ \theta coscos\ \theta\ \ +sinsin\ \theta\ -sinsin\ \theta coscos\ \theta\ \ -\theta\ -coscos\ \theta\ +sinsin\ \theta\ +coscos\ \theta\ +1}{(sin\theta+cos\theta)^2-1}$

$=\ \frac{\theta\ +2sinsin\ \theta\ -\theta\ +\theta\ +\theta}{2sinsin\ \theta coscos\ \theta}$

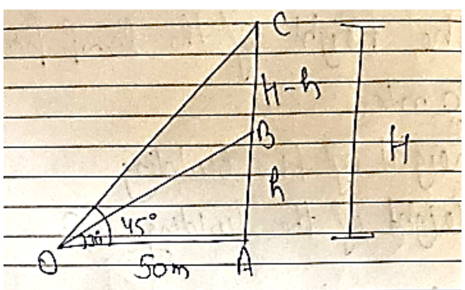
$= \frac{2sin^2\theta+2sin\theta}{2sin\theta cos\theta}$

$= \frac{sin\theta+1}{cos\theta}$

13. Answer any **one** question: 5

(i) Angle of elevation of the top of an incomplete tower from a point at distance $50\ m$ from its foot is 30°. How much should the height of the tower be increased so that the angle of elevation of the top will be 45° from that point?

Explanation:



To find height of $\triangle ABO$.

$tan30 = \frac{AB}{OA} = \frac{h}{50}$

$\frac{1}{\sqrt{3}} = \frac{h}{50}$

$\frac{50}{\sqrt{3}} = h$

In $\triangle AOC$

$tan45 = \frac{H}{50}$

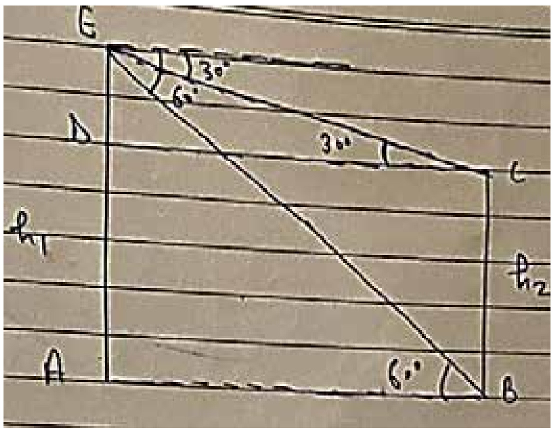
H = 50

Height of the of tower increased,

$\Rightarrow H - h \Rightarrow 50 - \frac{50}{\sqrt{3}} = 21.13m$

(ii) From the roof of the building the angle of depression of the top and foot of the lamp post are 30° and 60° respectively. Find the ratio of the heights of building and lamp post.

Explanation:



In $\triangle EDC$,

$$tan30^{\circ} = \frac{ED}{DC} \Rightarrow \frac{AE-AD}{CD}$$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AE-AD}{CD}$ .....(1)

Now, In $\triangle EAB$,

$$tan60^{\circ} = \frac{AE}{AB} \quad \Rightarrow \sqrt{3} = \frac{AE}{CD} - \ (2)$$

Divide (1) by (2),

$$\frac{AE-AD}{AE} = \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{AE}{AE} - \frac{AD}{AE} = \frac{1}{3}$$

$$1 - \frac{1}{3} = \frac{AD}{AE}$$

$$\Rightarrow \frac{2}{3} = \frac{AD}{AE} \Rightarrow \frac{BC}{AE} = \frac{2}{3}$$

$$\Rightarrow \frac{AE}{BC} = \frac{3}{2}$$

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14. Answer any **two** questions: 4 x 2 = 8

(i) Two solid spheres with the radii of $1\ cm$ and $6\ cm$ lengths are melted and a hollow sphere with outer radius $9\ cm$ is made. Determine the inner radius of the new hollow sphere.

Answer: 8 cm

Explanation:

Volume of a sphere $= \frac{4}{3}\pi r^3$

Total volume of two spheres $= \frac{4}{3}\pi\left(1^3 + 6^3\right)$

Let internal radius of hollow sphere $= rcm$ then

Volume of the iron of this sphere $= \frac{4}{3}\pi\left(9^3 - r^3\right)$ cu.cm

According to the question,

$$\frac{4}{3}\pi\left(1^3 + 6^3\right) = \frac{4}{3}\pi\left(9^3 - r^3\right)$$

$$1 + 6^3 = 9^3 - r^3$$

$$r^3 = 9^3 - 6^3 - 1 = 729 - 216 - 1 = 512$$

$$\Rightarrow r = \sqrt[3]{512} = 8\ cm$$

(ii) The height of a right circular cone is twice its radius of base. If the height be seven times of the diameter of the base then volume of the cone would have been $539 cucm$ more. Find the height of the cone.

Answer: 7 cm

Explanation:

Let the radius of cylinder $= r$ and the height of cylinder $= 2r$

If its height be 7 times its diameter, new height of cylinder $= 14r$

Solution: 1^{st} Case

Radius = r, Height = 2r

$Volume \ = \frac{1}{3}2\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times r^2 \times 2r$

$= \frac{44r^3}{21}$

2^{nd} Case

Radius = r, Height = 14r

Volume $= \frac{1}{3} \times \ \pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times r^2 \times 14r$

$= \frac{308r^3}{21}$

Volume of the cylinder $= \frac{1}{3} \times \ \pi r^2 \times 2r \ = \frac{1}{3} \times 2\pi r^3$

If $h \ = \ 14r$, volume $= \frac{1}{3} \times \ \pi r^2 \times 14r \ = \frac{1}{3} \times 14\pi r^3$

According to the Question,

$\frac{14}{3} \times \ \pi r^3 - \frac{2}{3} \times \ \pi r^2 \ = \ 539$

or. $4\pi r^3 \ = \ 539$

$or, \ 4 \times \frac{22}{7} \times r^3 \ = \ 539$

$or, \ r^3 \ = \ 539 \times \frac{1}{4} \times \frac{7}{22}$

or, $r^3 \ \ = \frac{7\times7\times7}{2\times2\times2} \ r \ \ = \frac{7}{2} \therefore 2r \ \ = \ 7\ r \ \ = \frac{7}{2}$

Given Height is twice of the radius,

H = r ×2

H = $\frac{7}{2}$ ×2

H = 7 cm.

(iii) Curved surface area of a right circular cylindrical wooden log of uniform density is 440 sq. decimeter. Weight of 1 cubic decimeter of wood is $3\ kg$ and weight of the log is 18.48 quintal. Find the diameter of the log.

Answer: $5.6\ dcm$

Explanation:

$$1\ quintals\ = 100\ kg$$
$$\therefore 18.48\ quintals\ = 18.48 \times 100 = 1848\ kg$$

Weight of 1 cubic decimeter of wood is $3\ kg$

$$3\ kg\ of\ wood = 1\ dcm^3$$
$$1\ kg\ of\ wood = \frac{1}{3}\ dcm^3$$
$$1848\ kg\ of\ wood = \frac{1848}{3} = 616\ dcm^3$$
$$Surface\ area = 2\pi r \times h = 440 dcm^2$$
$$Volume = \pi r^2 h = 616 dcm^3$$

By condition, $2\pi r h = 440$

$$h = \frac{440}{2\pi r} = \frac{440}{2r} \times \frac{7}{22} = \frac{70}{r}$$

Putting the value of ' h ' in volume we get

$$\pi r^2 h = 616$$
$$\Rightarrow \pi r^2 \frac{70}{r} = 616$$
$$\Rightarrow \frac{22}{7} \times r \times 70 = 616$$
$$\Rightarrow r = \frac{616}{220} = 2.8\ dcm$$

Diameter of the log $= 2r = 2 \times 2.8 = 5.6\ dcm$

15. Answer any **two** questions: 4 x 2 = 8

(i). If the arithmetic mean and total frequency of the following distribution are 50 and 120 respectively then find the value of f_1 and f_2:

Class	0 - 20	20 - 40	60 - 80	80 - 100	0 - 20
Frequency	17	f_1	32	f_2	19

Answer: $f_1 = 28$, $f_2 = 24$

Explanation:

classes	mid value (x_1)	frequency (f_1)	$f_i x_i$
$0-20$	10	17	170
$20-40$	30	f_1	$30f_1$
$40-60$	50	32	1600
$60-80$	70	f_2	$70f_2$
$80-100$	90	19	1710
		$68+f_1+f_2$	$3480+30f_1+70f_2$

$$\Sigma f_i = 120$$

$$\therefore we\ have\ 120 = 68 + f_1 + f_2$$

$$\Rightarrow f_1 + f_2 = 52 ....(1)$$

$$mean = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$50 = \frac{3480++30f_i+70f_2}{120}$$

$$30f_1 + 70f_2 = 6000 - 3460 = 25203f_1 + 7f_2 = 252...(2)$$

$eq(1)\times7 \Rightarrow 7f_1 + 7f_2 = 7\times52 = 364....(3)$

Subtract eq (3) from eq (2)

$$\Rightarrow 7f_1 + 7f_2 - 3f_1 - 7f_2 = 364 - 252$$

$$\Rightarrow 4f_1 = 112 \Rightarrow f_1 = 28$$

Substitute f_1 in eq (1)

$$f_2 = 52 - 28 = 24$$

Hence, the values of f_1 and f_2 are 28 and 24.

(ii) Construct the table of cumulative frequency (greater than type) and draw the ogive from the following frequency distribution:

Class	$0 - 10$	$10 - 20$	$20 - 30$	$30 - 40$	$40 - 50$	$50 - 60$
Frequency	7	10	23	50	6	4

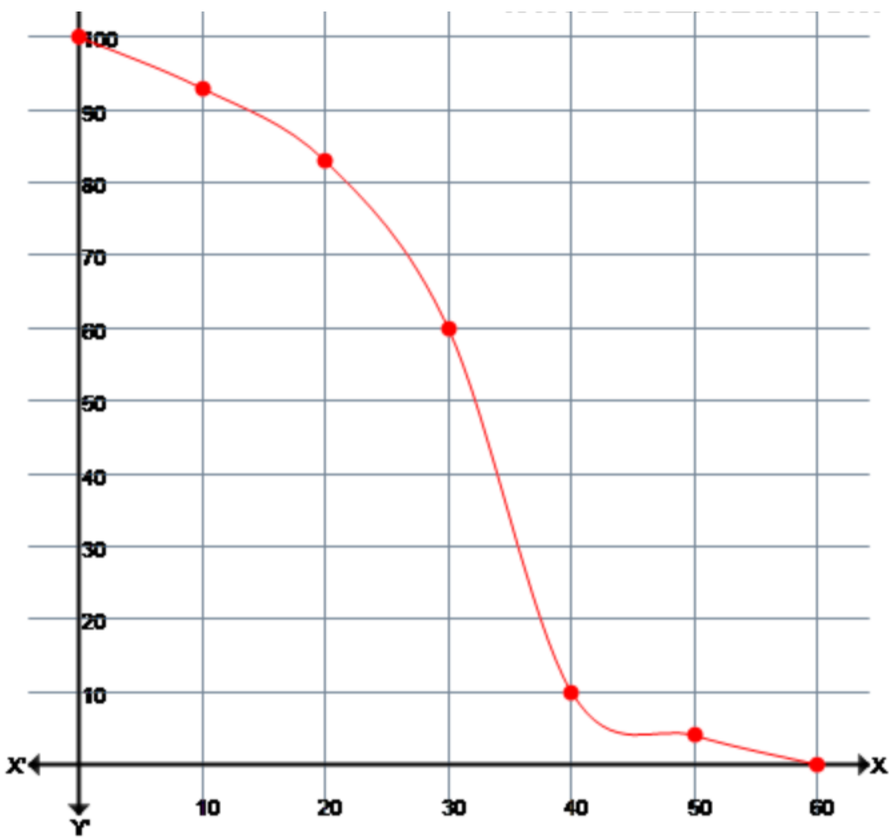
Explanation:

To draw More than type cumulative frequency curve, we first prepare the following table

Class	Greater than type c.f.	Frequency	cf
$0 - 10$	Greater than 0	7	100
$10 - 20$	Greater than 10	10	93
$20 - 30$	Greater than 20	23	83
$30 - 40$	Greater than 30	50	60
$40 - 50$	Greater than 40	6	10
$50 - 60$	Greater than 50	4	4

$60 - 70$	Greater than 60	–	0

Now, plot the cumulative frequencies against the lower limit of the class intervals. The curve obtained on joining the points so plotted is known as the more than ogive.



(iii) Find the mode of the following frequency distribution:

Class	$50 - 59$	$60 - 69$	$70 - 79$	$80 - 89$	$90 - 99$	$100 - 109$
Frequency	5	20	40	50	30	6

Answer: $82.8\overline{3}$

Explanation:

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CLASS	CLASS	frequency
50-59	$49.5 - 59.5$	5
60-69	$59.5 - 69.5$	20
70-79	$69.5 - 79.5$	40
80-89	$79.5 - 89.5$	5.0
90-99	$89.5 - 99.5$	30
100-109	$99.5 - 109.5$	6

Mode of the given data -

$$Mode = l + \left(\frac{f_1 - f_0}{2f_1 - t_0 - f_2}\right) \times h$$

$l = 79.5\ h = 10\ f_1 = 50\ f_0 = 40\ f_2 = 30$

Mode $= 79.5 + \left(\frac{50-40}{2\times50-40-30}\right)\times10$

$= 79.5 + \frac{10}{30}\times10$

$= 79.5 + 3.3\overline{3}$

$= 82.8\overline{3}$

[Alternative Question for Sightless Candidates]

11. Answer any **one** question: 5

(i) Describe the process of drawing an incircle of a right angled triangle.

To draw an incircle of a right-angled triangle, follow these steps:

1 Draw a right-angled triangle: Start by drawing a triangle with one angle of 90 degrees, which means that one of the sides must be vertical, and another horizontal.

2 Find the Incenter: To draw the incircle, you first need to find the incenter of the triangle. The incenter is the point where the angle bisectors of the triangle intersect. To find

the incenter, draw the angle bisectors of any two angles of the triangle. These two lines will intersect at a single point, which is the incenter.

3 Draw the Incircle: With the incenter located, you can now draw the incircle. The incircle is a circle that is tangent to all three sides of the triangle. To draw the incircle, use a compass to draw a circle with the incenter as the center, and a radius equal to the distance between the incenter and any of the sides of the triangle.

4 Check your work: Once you've drawn the incircle, check to make sure that it is indeed tangent to all three sides of the triangle. If it is not, check your measurements and make any necessary adjustments.

That's it! You have successfully drawn an incircle of a right-angled triangle.

(ii) Describe the method of construction of a square of equal area of an equilateral triangle

Solution:

To construct a square of equal area to an equilateral triangle, follow these steps:

1 Draw an equilateral triangle: Start by drawing an equilateral triangle of any size you want. To draw an equilateral triangle, use a compass to draw a circle of the desired size, then use a straightedge to draw a line through the center of the circle to create the first side of the triangle. Then, use the compass to draw the other two sides of the triangle, ensuring that all three sides are equal in length.

2 Find the height of the equilateral triangle: The height of an equilateral triangle is a line drawn from any vertex of the triangle perpendicular to the opposite side. To find the height, use a compass to draw a circle with the center at any vertex of the triangle and a radius equal to the length of one side of the triangle. Then, draw a line from that vertex to the point where the circle intersects the opposite side. This line will be the height of the equilateral triangle.

3 Draw a square with the same height: Now that you have found the height of the equilateral triangle, use a straightedge and compass to draw a square with the same height as the triangle. To do this, draw a line perpendicular to the height of the triangle at the point where it intersects the base of the triangle. This line will be the side of the square. Then, use the compass to draw the other three sides of the square.

4 Check your work: Once you've drawn the square, check to make sure that it has the same area as the equilateral triangle. To do this, you can calculate the area of the equilateral triangle using the formula $A = (sqrt(3)/4)^{*} s^{\wedge}2$, where A is the area and s is the length of one side. Then, calculate the area of the square using the formula $A = s^{\wedge}2$, where A is the area and s is the length of one side. If the two areas are equal, then you have successfully constructed a square of equal area to the equilateral triangle.

That's it! You have successfully constructed a square of equal area to an equilateral triangle.

[Additional Question for External Candidates]

16. (a) Answer any **three** questions: 2 × 3 = 6

(i) If $x \propto y, y \propto z$ and $z \propto x$, then find the relation between the constants of variations.

Explanation:

Given: $x \propto y, y \propto z$ and $z \propto x$
It can be written as, $x = ky, y = mz \; and \; z = nx$

where k, m and n are constant of variations.
Put the value of y in x
We get, $x = kmz$

Now put the value of z in x,

$\Rightarrow x = kmnx$
$\Rightarrow kmn = 1$

This is the required relation among three constants of variation i.e. k, m and n.

(ii) In a partnership business the capital of A is $1\frac{1}{2}$ times that of B. At the end of the year if B gets Rs. 1,500 as share of profit, find the share of A.

Answer: Rs. 2250

Explanation:

Let B's capital is x, then A's capital is $\frac{3}{2}x$

According to the question, B gets Rs. 1,500 as share of profit i.e. x = 1500

A's capital = $\frac{3}{2}x = \frac{3}{2}\times 1500 = 2250$

Hence, the share of A is Rs. 2250

(iii) If $x + \sqrt{x^2 - 9} = 9$ then find the value of $x - \sqrt{x^2 - 9}$.

Answer: $\frac{9}{4}$

Explanation:

Given, $x + \sqrt{x^2 - 9} = 4$

$$\frac{1}{x+\sqrt{x^2-9}} = \frac{1}{4}$$

$$\frac{x-\sqrt{x^2-9}}{(x)^2-\left(\sqrt{x^2-9}\right)^2} = \frac{1}{4}$$

$$\frac{x-\sqrt{x^2-9}}{9} = \frac{1}{4}$$

$$x - \sqrt{x^2 - 9} = \frac{9}{4}$$

Hence, the value of $x - \sqrt{x^2 - 9}$ is $\frac{9}{4}$

(iv) The numerical value of volume of a sphere is twice the numerical value of its surface area. Find the radius of the sphere.

Answer: 6 cm

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Explanation:

The volume of a sphere is twice its surface area.

The volume of a sphere $= (4/3)\pi r^3$

Surface Area of a sphere $= 4\pi r^2$

where " r " is the radius of the circle.

According to the question,

The volume of the sphere = Twice the surface area of the sphere

$$(4/3)\pi r^3 = 2\times\left(4\pi r^2\right)$$
$$\Rightarrow (4/3)r = 8$$
$$\Rightarrow r = 6$$

∴ The value of the radius of the sphere is $6\ cm$.

16. (b) Answer any **four** questions : 1 × 4 = 4

(i) Which one is greater $\sqrt{7}-\sqrt{2}$ or $\sqrt{8}-\sqrt{3}$?

Answer: $\sqrt{7}-\sqrt{2}$

Explanation:

Let $x = \sqrt{7}-\sqrt{2}$ & $y = \sqrt{8}-\sqrt{3}$

$$\frac{1}{x} = \frac{1}{\sqrt{7}-\sqrt{2}} \times \frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}+2} = \frac{\sqrt{7}+\sqrt{2}}{7-2} = \frac{\sqrt{7}+\sqrt{2}}{5} \quad \frac{1}{y} = \frac{1}{\sqrt{8}-\sqrt{3}} \times \frac{\sqrt{8}+\sqrt{3}}{\sqrt{8}+\sqrt{3}} = \frac{\sqrt{8}+\sqrt{3}}{8-3} = \frac{\sqrt{8}+\sqrt{3}}{5} \quad clearly \quad \frac{1}{x} < \frac{1}{y}$$

Therefore $\sqrt{7}-\sqrt{2}$ is greater

(ii). Under which condition the quadratic equation $ax^2 + bx + c = 0(a\neq 0)$ has one zero root.

Answer: $c = 0,\ a\neq 0,\ b\neq 0$

Explanation:

One zero means x=0

$$ax^2 + bx + c = 0$$

$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \sqrt{b^2 - 4ac} = 0$

$\Rightarrow ac = 0$ i.e a=0 and c=0

If a=0, then x=-c/b which is not equals to zero

If c=0, $ax^2 + bx = 0$

$x(ax + b) = 0$

Therefore, x = 0

So, the condition is , $c = 0,\ a \neq 0,\ b \neq 0$

(iii). If the lengths of three sides of two triangles are in proportion, then which type of triangle is this?

Answer: Similar triangle

Explanation:

If all the three sides of a triangle are in proportion to the three sides of another triangle, then the two triangles are similar. So, it is a similar triangle.

(iv). In how many years a sum of money at $6\frac{1}{4}$ % simple interest per annum would be double?

Answer: 16 years

Explanation:

Given, $R = 6\frac{1}{4} = \frac{25}{4}$ %

According to the question, Amount $= 2$ (Principle)

$A = 2P$

$S.I. = A - P \quad = 2P - P \quad = P$

$S.I. \ = \frac{P \times R \times T}{100}$

$P = \frac{P \times 25 \times T}{4 \times 100}$

$T = \frac{P \times 4 \times 100}{P \times 25}$

$T = 16\ years$

(v) The front formed at the centre of a circle by an arc, is the _____________ of the angle formed by the same arc at point on the circle.

Answer: double

Explanation: The front formed at the centre of a circle by an arc, is the **double** of the angle formed by the same arc at point on the circle.