Madhyamik Maths Question Paper 2019 With Solutions

Question 1: Choose the correct option in each case from the following questions : [1 x 6 = 6]

[i] In a partnership business, the ratio of the share of profit of two friends is 1 / 2:1 / 3, then the ratio of their principal is -

(a) 2 : 3 (b) 3 : 2 (c) 1 : 1 (d) 5 : 3

Answer: (b)
1 / 2:1 / 3
3 / 6:2 / 6
3:2

[ii] If p + q = √13 and p − q = √5 then the value of pq is −

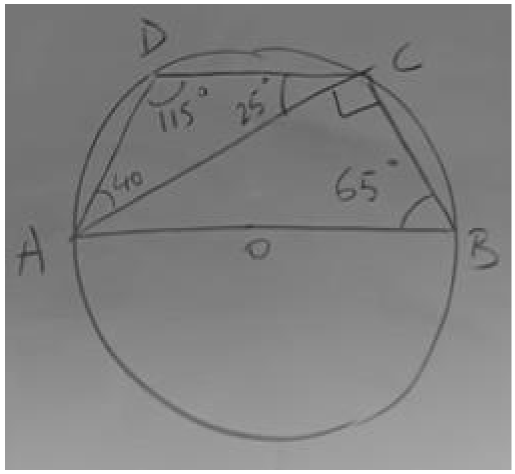
(a) 2 (b) 18 (c) 9 (d) 8

Answer: (a)
$p^2 + q^2 + 2pq = 13$
$p^2 + q^2 - 2pq = 5$
$4pq = 8$
$pq = 2$

[iii] O is the centre of a circle and AB a diameter. ABCD is a cyclic quadrilateral. ∠ABC = 65°, ∠DAC = 40°, then the measure of ∠BCD is −

(a) 75° (b) 105° (c) 115° (d) 80°

Answer: (c)



$\angle BCD = 25 + 90 = 115^{\circ}$

[iv] If $\tan\alpha + \cot\alpha = 2$, then the value of $\tan^{13}\alpha + \cot^{13}\alpha$ is -

(a) 13 (b) 2 (c) 1 (d) 0

Answer: (b)

$\tan\alpha + 1 / \tan\alpha = 2$

$\tan^2\alpha + 1 - 2\tan\alpha = 0$

$(\tan\alpha - 1)^2 = 0$

$\tan\alpha = 1$

$\tan^{13}\alpha + \cot^{13}\alpha$

$= 1 + 1$

$= 2$

[v] If two cubes of the length of each side $2\sqrt{6}$ cm are placed side by side, then the length of the diagonal of the cuboid so produced is –

(a) 10cm (b) 6cm (c) 2cm (d) 12cm

Answer: (d)

Diagonal = $\sqrt{l^2 + b^2 + h^2}$
$= \sqrt{(4\sqrt{6})^2 + (2\sqrt{6})^2 + (2\sqrt{6})^2}$
$= \sqrt{144}$
$= 12$

[vi] The mean of the data $x_1, x_2, x_3 \ldots\ldots x_{10}$ is 20 then the mean of $x_1 + 4, x_2 + 4, x_3 + 4, \ldots\ldots\ldots x_{10} + 4$ will be –

(a) 20 (b) 24 (c) 40 (d) 10

Answer: (b)

Question 2: Fill up the blanks (any five) : [1 x 5 = 5]

(i) A person deposited Rs. 100 in a bank and gets the amount Rs. 121 after two years. The rate of compound interest is _______ %. **[Answer: 10%]**
P = 100, A = 121, T = 2 years, R = 9
$A = P [1 + R / 100]^n$
$121 = 100 [1 + R / 100]^2$
$[11 / 10]^2 = [1 + R / 100]^2$
R = 10%

(ii) If the product and sum of two quadratic surds is a rational number, then surds are _______ surd. **[Answer: Conjugate]**

(iii) If the bases of two triangles are situated on the same line and the other vertex of the two triangles are common, then the ratio of the areas of two triangles are _______ to the ratio of their bases. **[Answer: equal]**

(iv) The number of surfaces of a solid right circular cylinder is _______. **[Answer: 3]**

(v) The simplest value of $\cos 53^\circ / \sin 37^\circ$ is _______. **[Answer: 1]**
$\cos 53^\circ / \sin 37^\circ$

= sin [90 - 53] / sin 37
= sin 37 / sin 37
= 1

(vi) The variables x_1, x_2, ….. x_{100} are in ascending order of their magnitude, then the median of the variables is _______. **[Answer: $(x_{50} + x_{51}) / 2$]**

Question 3: Write True or False (any five): [1 x 5 = 5]

(i) The difference between the simple interest and the compound interest of Rs. 100 in 1 year at the rate of 10% p.a. is Re. 1. **[Answer: False]**

(ii) The compound ratio of $ab:c^2$, bc:a and $ca:b^2$ is 1:1. **[Answer: True]**

(iii) Only one circle can be drawn through three noncollinear points. **[Answer: True]**

(iv) sin 30º + sin 60º > sin 90º. **[Answer: True]**

(v) The ratio of the volume of a right circular cone and a right circular cylinder with the same base and height is 1:3. **[Answer: True]**

(vi) Value of the median of data 2, 3, 9, 10, 9, 3, 9 is 10. **[Answer: False]**

Question 4: Answer the following questions (any ten) : [2 x 10 = 20]

(i) Find the capital which gives Re. 1 as interest per month at 5% rate of interest per annum.

Solution:
SI = Re.1
R = 5%
T = 1 month = 1 / 12 years
P = ?
SI = PTR / 100

$1 = P * (1 / 12) * 5 / 100$
$100 = P (5 / 12)$
P = Rs.240

(ii) In a partnership business, the ratio of capitals of three men is 3:5:8. The share of profit of the first member is Rs. 60 less than that of the third member, then what is the total profit in the business?

Solution:
The profit will be shared among the partners in their capital ratio.
Let the total profit be X.
1^{st} person profit will be 3X.
2^{nd} person profit will be 5X.
3^{rd} person profit will be 8X.
$3X = 8X - 60$
$60 = 5X$
$X = 60 / 5$
$X = 12$
Total profit = $3X + 5X + 8X = 16X$
Rs. 16 * 12 = Rs. 192 is the total profit.

(iii) If $a / 2 = b / 3 = c / 4 = [2a - 3b + 4c] / p$, then find p.

Solution:
Let $a / 2 = b / 3 = c / 4 = 2a - 3b + 4c / k = m$
So $a = 2k$
$b = 3k$
$c = 4k$
$2a - 3b + 4c = pk$
$2 * 2k - 3 * 3k + 4 * 4k = pk$
$11k = pk$
$p = 11$

(iv) $x \alpha y^2$ and $y = 2a$ when $x = a$, then show that $y^2 = 4ax$.

Solution:

$x \alpha y^2$

$x = ky^2$ ---- (1)

$a = k * (2a)^2$

$a = 4a^2k$

$k = 1 / 4a$

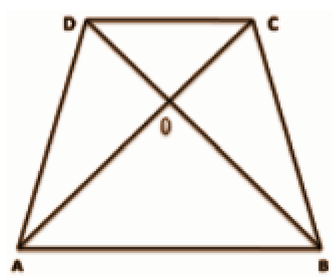
Put the value of k in (1)

$x = (1 / 4a) * y^2$

$4ax = y^2$

(v) In a trapezium ABCD, BC || AD and AD = 4 cm. The two diagonals AC and BD intersect at the point O in such a way that AO / OC = DO / OB = 1 / 2. Calculate the length of BC.

Solution:



ABCD is a trapezium such that BC || AD and AD = 4 cm.

If the diagonals AC and BD intersect at O such that AO / OC = DO / OB = 1 / 2

Now from the triangle ΔAOB and ΔCOD,

∠AOB = COD (vertically opposite angle)

AO / OC = DO / OB (Given)

From SAS congruence criterion,

ΔAOB ≅ ΔCOD

AO / OC = BO / OD = AB / DC (since corresponding sides of similar triangles are proportional)

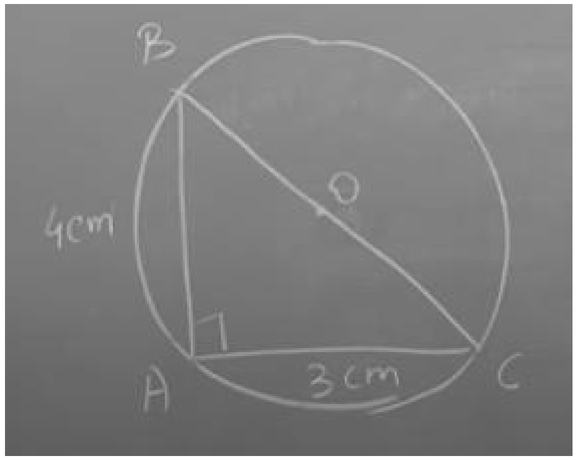
=> 1 / 2 = 4 / DC

=> DC = 4 * 2

=> DC = 8 cm

(vi) Two chords AB and AC of a circle are mutually perpendicular to each other. If AB = 4 cm and AC = 3 cm, find the length of the radius of the circle.

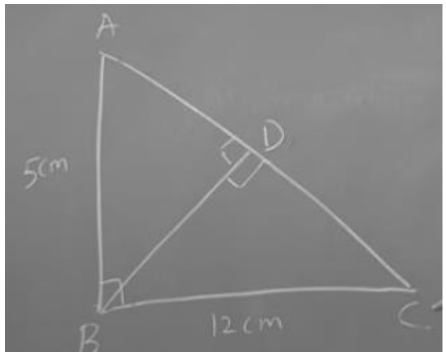
Solution:



The angle of a semicircle is 90^o.
BAC is the semi-circle in the figure.
BC is the diameter.
Triangle ABC is a right-angled triangle.
By applying the Pythagoras theorem,
$AB^2 + AC^2 = BC^2$
$4^2 + 3^2 = BC^2$
$BC = \sqrt{16 + 9} = \sqrt{25}$
BC = 5cm
Radius = Diameter / 2
= 5 / 2
= 2.5cm

(vii) In $\triangle ABC$, $\angle ABC = 90^o$ and $BD \perp AC$, if AB = 5cm, BC = 12 cm, then find the length of BD.

Solution:



In $\triangle ABC$,

$AB^2 + BC^2 = AC^2$

$5^2 + 12^2 = AC^2$

$25 + 144 = AC^2$

$AC = 13cm$

Let $BD = y$, $AD = x$

$CD = 13 - x$

In $\triangle ADB$,

$x^2 + y^2 = 5^2$

$y^2 = 25 - x^2$ ---- (1)

In $\triangle BDC$,

$(13 - x)^2 + y^2 = 12^2$ ---- (2)

$(13 - x)^2 + (25 - x^2) = 144$

$13^2 + x^2 - 26x + 25 - x^2 = 144$

$169 - 26x + 25 = 144$

$194 - 144 = 26x$

$50 / 26 = x$

$x = 25 / 13$

$y^2 = 25 - (25 / 13)^2$

$= 25 - [625 / 169]$

$= 3600 / 169$

$BD = y = 60 / 13$ cm

(viii) Find the value(s) of Θ ($0^o \leq \Theta \leq 90^o$) for which $2 \sin \Theta \cos \Theta = \cos \Theta$.

Solution:

$2 \sin \Theta \cos \Theta = \cos \Theta$
$2 \sin \Theta = 1$
$\sin \Theta = 1 / 2$
$\Theta = 30^{\circ}$

(ix) If sin 10Θ = cos 8 Θand 10Θ is a positive acute angle, then find the value of tan 9Θ.

Solution:
$\sin 10\Theta^{\circ} = \cos 8\Theta^{\circ}$
$\sin 10\Theta^{\circ} = \sin (90 - 8\Theta)^{\circ}$
Comparing both sides:
$10\Theta = 90 - 8\Theta$
$18\Theta = 90$
$\Theta = 90 / 18$
$\Theta = 5^{\circ}$
Therefore,
$\tan 9\Theta^{\circ} = \tan 9 \times 5^{\circ}$
$= \tan 45^{\circ}$
$= 1$

(x) The length, breadth and height of a cuboidal room be a unit, b unit, and c unit respectively and a + b + c = 25, ab + bc + ca = 240.5 then find the length of the longest rod to be kept inside the room.

Solution:
The diagonal of the cuboidal = $\sqrt{a^2 + b^2 + c^2}$ units
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
$\Rightarrow (25)^2 = (a^2 + b^2 + c^2) + 2 (240.5)$ {Since, $(a + b + c) = 25$ and $(ab + bc + ca) = 240.5$. given}
$\Rightarrow (a^2 + b^2 + c^2) = 144$
$\Rightarrow \sqrt{a^2 + b^2 + c^2}$
$= \sqrt{144}$
$= 12$ units

(xi) The area of the curved surface of a right circular cone is √5 times that of the base of the cone, find the ratio of the height and the radius of the base of the cone.

Solution:

The area of the curved surface of a right circular cone is √5 times its area of the base.

Area of the curved surface of a right circular cone = πRL

R Radius of Base

L = Slant Height

Area of Base = πR^2

Area of the curved surface of a right circular cone = √5 * Area of Base

=> $\pi RL = \sqrt{5}\,\pi R^2$

=> $L = \sqrt{5}\,R$

$L^2 = H^2 + R^2$ (H = Height of cone)

=> $(\sqrt{5}\,R)^2 = H^2 + R^2$

=> $5R^2 = H^2 + R^2$

=> $H^2 = 4R^2$

=> $H = 2R$

The height of the cone is 2 times its radius.

(xii) The mid-value of the first (2n + 1) consecutive natural number is [n + 103] / 3. Find n.

Solution:

The first (2n + 1) consecutive natural numbers form an arithmetic progression.

1, 2, 3, 4.................... (2n + 1)

As (2n + 1) is odd number, the middle term will be (2n + 2) / 2 = $(n + 1)^{th}$ term

Now, value of T_{n+1} term is given by

= 1 + (n + 1 - 1)*1

= n + 1

$n + 1 = (n + 103) / 3$
$3n + 3 = n + 103$
$3n - n = 103 - 3$
$2n = 100$
$n = 100 / 2 = 50$
Therefore the value of $n = 50$.

Question 5: Answer any one question : [5 x 1 = 5]

(i) If interest is compounded half-yearly what will be the compound interest and amount on Rs. 8,000 at the rate of 10% compound interest per annum for 1 (1 / 2) years?

(ii) Two friends start a partnership business investing Rs. 40,000 and Rs. 50,000 respectively. There is an agreement between them that 50% of the profit will be divided equally and the rest amount of profit will be distributed between them in the ratio of their principal. If the share of profit of 1st friend is Rs. 800 less than that of the 2nd friend, find the share of profit of the 1st friend.

Solution:
[i] P = Rs. 8000
R = 10%
T = 1 (1 / 2) years = 3 / 2 years
Compounded half-yearly, R = 10 / 2 = 5%
T = 2 * (3 / 2) = 3 years
$A = P \{1 + (R / 100)\}^n$
$A = 8000 \{1 + (5 / 100)\}^3$
$= 8000 * (1.05)^3$
= Rs. 9261
CI = A - P
= 9261 - 8000
= Rs. 1261

[ii] Let say Total Profit Share = P

Profit Share of 1^{st} friend = (50 / 100)(P/2) + (40000 / 90000)(P / 2)
= P / 4 + 4P / 18
Profit Share of 2nd friend = (50 / 100)(P / 2) + (50000 / 90000)(P / 2)
= P / 4 + 5P / 18
P / 4 + 5P / 18 - (P / 4 + 4P / 18) = P / 18
P / 18 = 800
=> P = 14400 Rs
First friend share = 14400 / 4 + 4 * 14400 / 18
= 3600 + 3200
= 6800 Rs
2^{nd} friend share = 14400 / 4 + 5 * 14400 / 18
= 3600 + 4000
= 7600 Rs

Question 6: Solve any one question: [3 x 1 = 3]

(i) Determine the equation whose roots are the square of the roots of the equation $x^2 + x + 1 = 0$.

(ii) If the price of 1 dozen pens is reduced by Rs 6, then 3 more pens will be bought at Rs 30. calculate the price of one dozen pens before the reduction of price.

Solution:
[i] $x^2 + x + 1 = 0$
$ax^2 + bx + c = 0$
a = 1, b = 1, c = 1
$\alpha + \beta = -b / a = -1 / 1 = 1$
$\alpha \beta = c / a = 1 / 1 = 1$
$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2 \alpha \beta$
$(-1)^2 = \alpha^2 + \beta^2 + 2 * 1$
$-1 = \alpha^2 + \beta^2$ --- (1)
$\alpha^2 \beta^2 = 1$ ---- (2)
$x^2 - (\alpha^2 + \beta^2) x + \alpha^2 \beta^2$
$x^2 + x + 1 = 0$

[ii] Let the price of 1 dozen pens before reduction be x.
After the reduction, the price of 12 pens is x - 6.
Price of 15 pens = ([x - 6] / 12) * 15 ----- (1)
Price of 15 pens = Rs. 30 ---- (2)
([x - 6] / 12) * 15 = 30
[x - 6] / 12 = 2
x - 6 = 24
x = Rs. 30

Question 7: Answer any one question: **[3 x 1 = 3]**

(i) Simplify: [4√3 / 2 − √2] − [30 / 4√3 − √18] − [√18 / 3 − √12].

(ii) If [(1 / x) - (1 / y)] α [1 / x - y], then show that $x^2 + y^2$ α xy.

Solution:
[i] [1] = (4√3) / (2 − √2) = (4√3) (2 + √2) / (2 − √2) (2 + √2)
= (8√3 + 4√6) / 2
= 4√3 + 2√6 ····················(1)

[2] = 30 / (4√3 − √18)
= 30 * (4√3 + √18) / (4√3 − √18) (4 √3 + √18)
= (120√3 + 30 √18) / (48 − 18)
= 30 (4√3 + √18) / 30
= 4√3 + 3√2 ··················(2)

[3] = √18 / (3 − 2√3)
= √18 (3 + 2√3) / (3 − 2√3) (3 + 2√3)
= (3√18 + 2√54) / 9 − 12
= (9√2 + 6√6) / −3
= −3√2 −2√6 ··················(3)

Now by combining all 3 terms:

$(4\sqrt{3} + 2\sqrt{6}) — (4\sqrt{3} + 3\sqrt{2}) — (-3\sqrt{2} - 2\sqrt{6})$

$= 4\sqrt{3} + 2\sqrt{6} - 4\sqrt{3} - 3\sqrt{2} + 3\sqrt{2} + 2\sqrt{6}$

$= 2\sqrt{6} + 2\sqrt{6}$

$= 4\sqrt{6}$

[ii] [y - x] / xy = k * [1 / [x - y]]

(y - x) (x - y) = k* xy

xy - x^2 - y^2 - xy = k * xy

$x^2 + y^2$ = 2xy - kxy

$x^2 + y^2$ = (2 - k) xy

$x^2 + y^2$ α xy

Question 8: Answer any one question: [3 x 1 = 3]

(i) If (3x - 2y) : (x + 3y) = 5 : 6, then find the value of (2x + 5y) : (3x + 4y).

(ii) If [b + c - a] / [y + z - x] = [c + a - b] / [z + x - y] = [a + b - c] / [x + y - z], then prove that [a / x] = [b / y] = [c / z].

Solution:

[i] (3x - 2y) : (x + 3y) = 5 : 6

6 * (3x - 2y) = 5 * (x + 3y)

18x - 12y = 5x + 15y

18x - 5x = 15y + 12y

13x = 27y

x = [27 / 13] * y

(2x + 5y) / (3x + 4y)

= (2 * [27y / 13] + 5y) / (3 * [27y / 13] + 4y)

= ([54y / 13] + 5y) / [(81y / 13) + 4y]

= 119y / 133y

= 119 / 133

= 17 / 19

= 17:19

[ii] [b + c - a] = k [y + z - x] ---- (1)
[c + a - b] = k [z + x - y] ---- (2)
[a + b - c] = k [x + y - z] ---- (3)
Add (1), (2) and (3),
a + b + c = k (x + y + z) ---- (4)
Subtract (1) from (4),
2a = k (2x)
a / x = k
Similarly,
b / y = k
c / z = k
From the above equations,
[a / x] = [b / y] = [c / z]

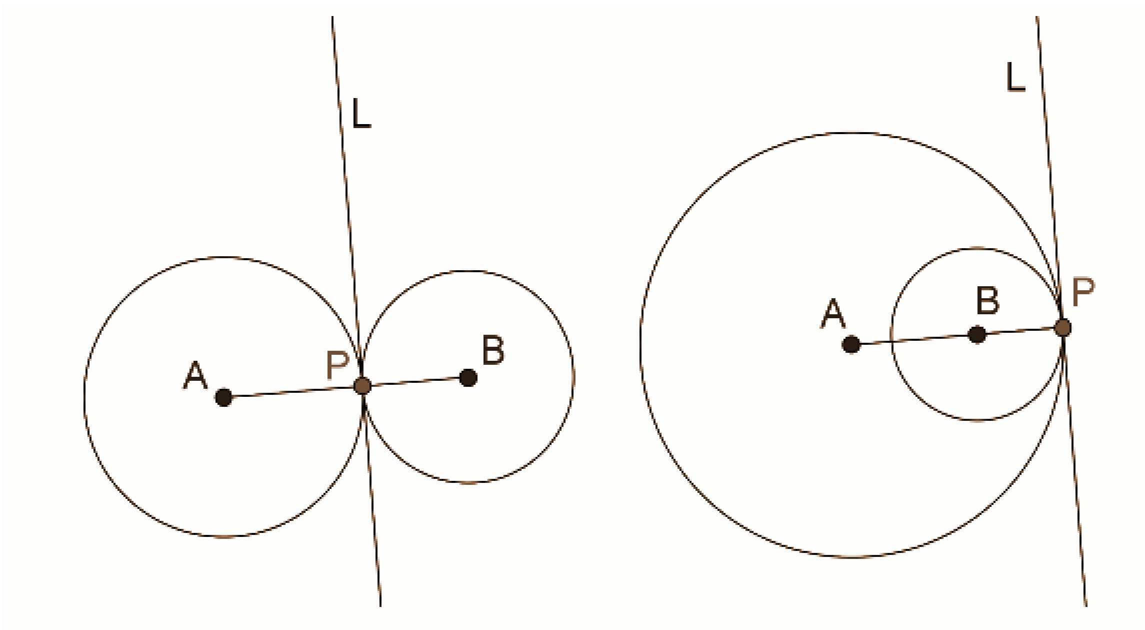
Question 9: Answer any one question: [5 x 1 = 5]

(i) Prove that the semicircular angle is a right angle.

(ii) If two circles touch each other externally then the point of contact will lie on the line segment joining the two centres - Prove it.

Solution:
[i] Now POQ is a straight line passing through centre O.
∴ Angle subtended by arc PQ at O is
$\angle POQ = 180^\circ$
The angle subtended by an arc at the centre is double the angle subtended by any point on the remaining part of the circle.
$\therefore \angle POQ = 2 \angle PAQ$
$\angle POQ / 2 = \angle PAQ$
$180 / 2 = \angle PAQ$
$\angle PAQ = 90^\circ$

[ii] Two circles are touching each other externally in the figure on the left side whereas, on the right side, two circles are touching each other internally. At the point of contact, a common tangent L can be drawn to both the circles:



In both the cases above, AP must be perpendicular to L, and so must BP. This is because L is the tangent to both the circles at P. Thus, both the perpendiculars AP and BP pass through the same point P, which necessarily implies that A, P and B are collinear.

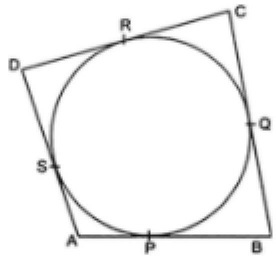
Question 10: Answer any one question: **[3 x 1 = 3]**

(i) If a quadrilateral ABCD is circumscribed about a circle with centre O, prove that AB + CD = BC + DA.

(ii) If in ΔABC, $\angle A$ is the right angle and BP and CQ are two medians, then prove that, $5BC^2 = 4(BP^2 + CQ^2)$.

Solution:

[i]



Since the tangents drawn from an exterior point to a circle are equal in length.

$\therefore$ AP = AS ...(i)

BP = BQ ...(ii)

CR = CQ ...(iii) and

DR = DS ...(iv)

Adding (i), (ii), (iii) and (iv),

AP + BP + CR + DR = AS + BQ + CQ + DS

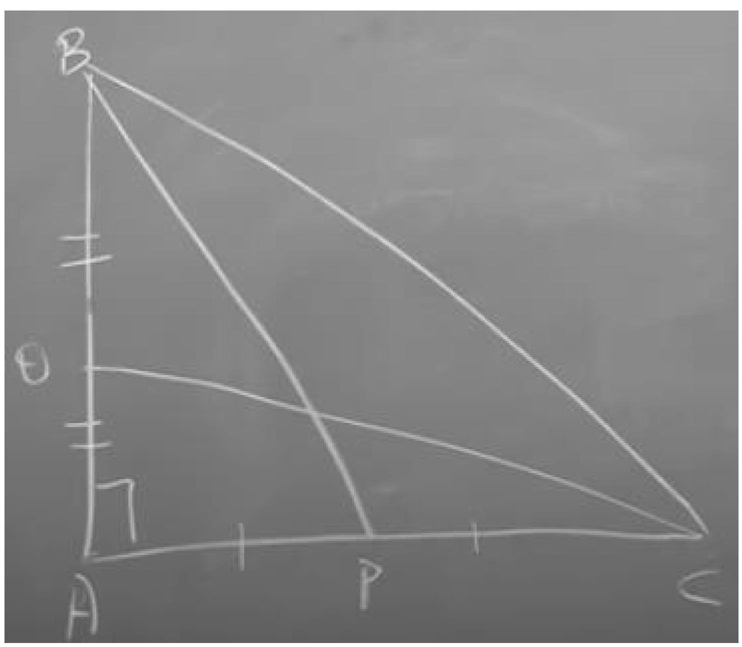
$\Rightarrow$ (AP + BP) + (CR + DR)

= (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC

Hence, AB + CD = BC + DA.

[ii]



In $\triangle$ABC,

$AB^2 + AC^2 = BC^2$ ---- (1)

In $\triangle$BAP,

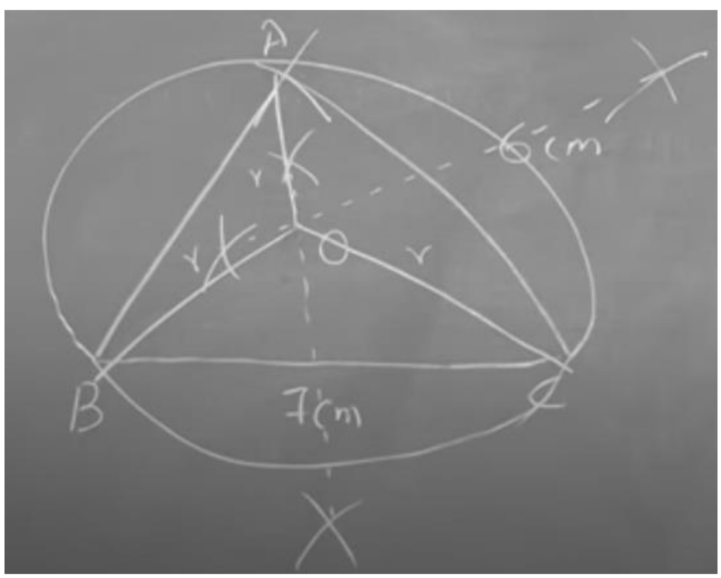
$AB^2 + AP^2 = BP^2$ ---- (2)
In $\triangle QAC$,
$QA^2 + AC^2 = QC^2$ ---- (3)
LHS = $5BC^2 = 5(AB^2 + AC^2)$
RHS = $4[AB^2 + (AC/2)^2 + AC^2 (AB/2)^2]$
$= 4[5(AB^2 + AC^2)/4]$
$= 5[AB^2 + AC^2]$

Question 11: Answer any one question: [5 x 1 = 5]

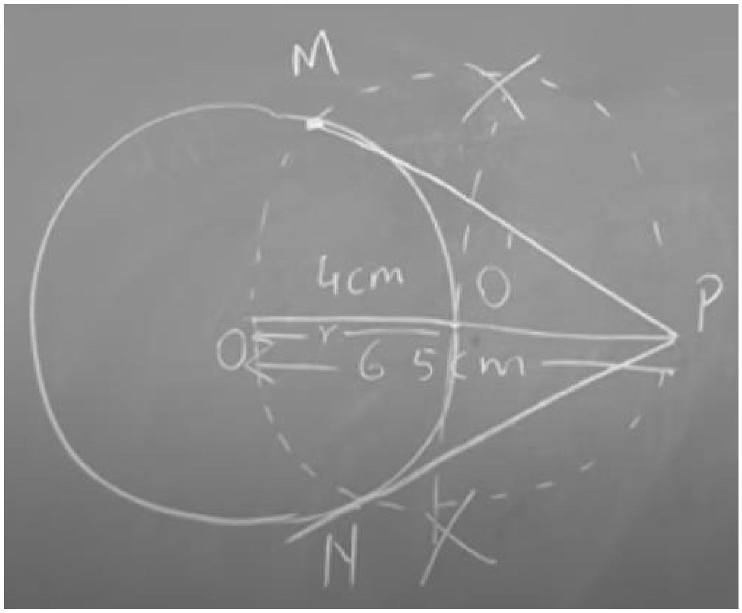
(i) Draw a triangle ABC of which BC = 7 cm, AB = 5 cm and AC = 6 cm. Then draw the circumcircle of ΔABC. (Only traces of construction are required)

(ii) Construct a circle of radius 4 cm and draw two tangents to the circle from an external point at a distance of 6.5 cm from the centre of the circle.

Solution:
[i]



[ii]



Question 12: Answer any two questions : **[3 x 2 = 6]**

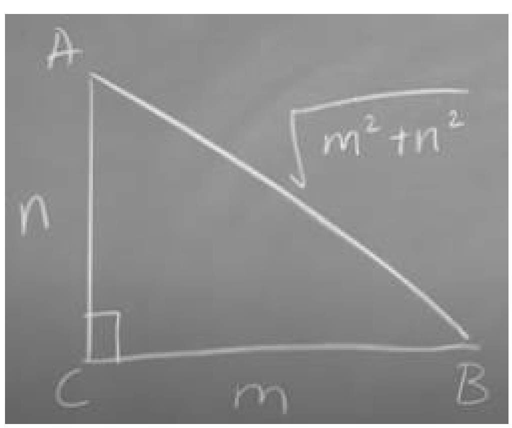
[i] In △ABC, ∠C = 90º, if BC = m and AC = n then prove that, m sin A + n sin B = $\sqrt{m^2 + n^2}$.

[ii] Find the value of (4 / 3) $\cot^2 30^\circ + 3\sin^2 60^\circ - 2\,\text{cosec}^2 60^\circ - (3/4)\tan^2 30^\circ$.

[iii] If ∠P + ∠Q = 90° then show that $\sqrt{\sin P / \cos Q} - (\sin P \cos Q) = \cos^2 P$.

Solution:

[i]



In $\triangle ACB$,

$AC^2 + BC^2 = AB^2$

$n^2 + m^2 = AB^2$

$AB = \sqrt{m^2 + n^2}$

LHS = m sin A + n sin B

$= m * (m / \sqrt{m^2 + n^2}) + n * (n / \sqrt{m^2 + n^2})$

$= m^2 / \sqrt{m^2 + n^2} + n^2 / \sqrt{m^2 + n^2}$

$= m^2 + n^2 / \sqrt{m^2 + n^2}$

$= \sqrt{m^2 + n^2} * \sqrt{m^2 + n^2} / \sqrt{m^2 + n^2}$

$= \sqrt{m^2 + n^2}$

= RHS

[ii] $(4 / 3) \cot^2 30^o + 3 \sin^2 60^o - 2 \operatorname{cosec}^2 60^o - (3 / 4) \tan^2 30^o$

$= (4 / 3) * (\sqrt{3})^2 + 3 * (\sqrt{3} / 2)^2 - 2 * (2 / \sqrt{3})^2 - (3 / 4) * (1 / \sqrt{3})^2$

$= (4 / 3) * (3) + 3 * (3 / 4) - 2 * (4 / 3) - (3 / 4) * (1 / 3)$

$= 4 + 9 / 4 - 8 / 3 - 3 / 12$

$= 10 / 3$

[iii] $\angle P + \angle Q = 90^\circ$

$90 - Q = P$

LHS = $\sqrt{\sin P / \cos Q - (\sin P \cos Q)}$

$= \sqrt{\sin P / \sin (90 - Q) - \sin P (\sin [90 - Q])}$

$= \sqrt{\sin P / \sin P - \sin P \sin P}$

$= 1 - \sin^2 P$

$= \cos^2 P$

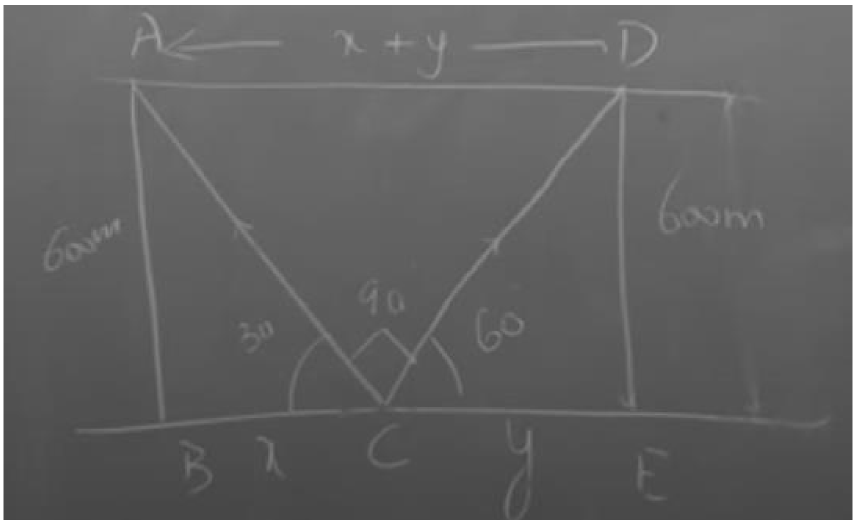
Question 13: Answer any one question: [5 x 1 = 5]

(i) From a quay of a river, 600 metres wide, two boats start in two different directions to reach the opposite side of the river. The first boat moves to make an angle of 30° with this bank and the second boat moves to make an angle 90° with the direction of the first boat. What will be the distance between the two boats when both of them reach the other side?

(ii) The length of the flag post at the roof of a three-storied building is 3.6 metre. The angles of elevation of the top and foot of the post are 50° and 45° respectively from a point on the road. Find the height of the building. [Take tan 50° = 1.2]

Solution:

[i]



In △ABC,

tan 30º = 600 / x

$1 / \sqrt{3} = 600 / x$

$x = 600\sqrt{3}$

In △DEC,

tan 60º = 600 / y

$\sqrt{3} = 600 / y$

$y = 600 / \sqrt{3}$

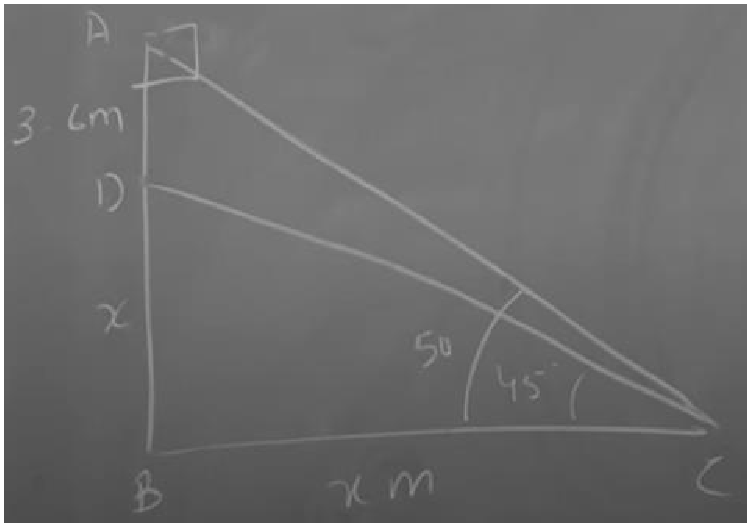
$x + y = 600\sqrt{3} + 600 / \sqrt{3}$

$= [1800 + 600] / \sqrt{3}$

$= 2400 / \sqrt{3}$

$= 800\sqrt{3}m$

[ii]



In $\triangle DBC$,

$DB / BC = \tan 45^\circ$

$x / BC = 1$

$BC = 'x'm$

In $\triangle ABC$,

$[3.6 + x] / x = \tan 50^\circ$

$[3.6 + x] / x = 1.2$

$3.6 + x = 1.2x$

$3.6 = 0.2x$

$x = 3.6 / 0.2$

$x = 18m$

Height of the building = 18m.

Question 14: Answer any two questions: [4 x 2 = 8]

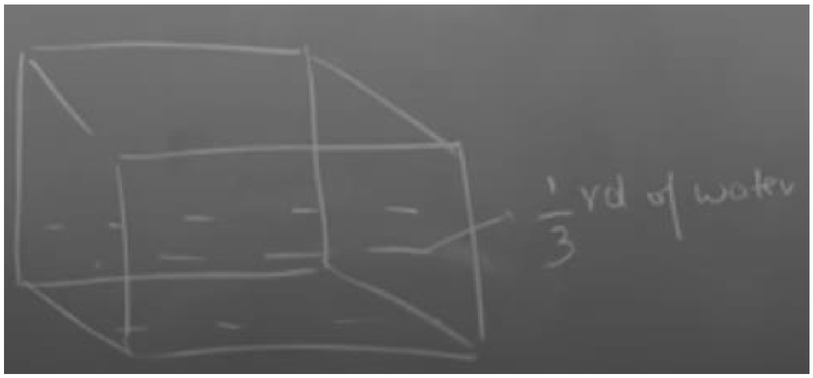
(i) If 64 buckets of water are withdrawn from a cubical water tank, full of water, then 1 / 3 of water in the tank still remains. If the length of the side of the water tank is 1.2 metre then what is the capacity (in litre) of each bucket? (1 cubic decimeter = 1 litre)

(ii) The diameter of the cross-section of a wire is reduced by 50%. If the volume remains constant, what percent of the length of the wire should be increased?

(iii) 77 sq. m tripal is required to make a right circular conical tent. If the slant height of the tent is 7 m, then what is the area of the base of the tent?

Solution:

[i]



The total volume of the cubical tank with a side (a) = a^3

Volume remained = $[1 / 3]a^3$

Volume emptied = $a^3 - [1 / 3]a^3$

The volume of 1 bucket = V

Volume emptied = 64 * volume of bucket

$[2 / 3]a^3 = 64 * V$

$(2 / 3) (12)^3 = 64 * V$

1152 / 64 = V

V = 18 / 1000 m^3

V = 18L

[ii] Let diameter = d units

Radius = r = d / 2

Volume $(V_1) = \boldsymbol{\pi} (d / 2)^2 * h_1$

Volume $(V_2) = \boldsymbol{\pi} (d / 4)^2 * h_2$

$V_1 = V_2$

$\boldsymbol{\pi} (d / 2)^2 * h_1 = \boldsymbol{\pi} (d / 4)^2 * h_2$

$4h_1 = h_2$

Increase % = [Increase / Original] * 100

$= [3h_1 / h_1] * 100$

= 300%

[iii] Structure of a tent is like a right circular cone.

∴ Tripal required to make tent = Curved surface area of the cone

Let, the base radius of the tent = r m

Slant height = 7 m

$\Rightarrow \pi \times r \times 7 = 77$

$\Rightarrow 22 / 7 \times r = 11$

$\Rightarrow r = 7 / 2 = 3.5$

∴ Base radius of the tent = 3.5 m

∴ Base area of the tent,= πr^2

$= 22 / 7 \times 3.5^2$

= 38.5 sq. m

Question 15: Answer any two questions: **[4 x 2 = 8]**

(i) If the arithmetic mean of the following frequency distribution is 54, then find the value of K:

Class	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
Frequency	7	11	K	9	13

(ii) Making frequency distribution table from the given cumulative frequency distribution table, find the mode of the data :

Class	<10	<20	<30	<40	<50	<60	<70	<80
Frequency	4	16	40	76	96	112	120	125

(iii) Find the mean of 52 students of class X in a school by using the direct method and assumed mean method from the table given below:

Class	**4**	**7**	**10**	**15**	**8**	**5**	**3**
Frequency	**30**	**33**	**35**	**40**	**43**	**45**	**48**

Solution:

[i]

Class	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
Frequency	7	11	K	9	13
x_i	10	30	50	70	90
$f_i x_i$	70	330	50K	630	1170

Mean = $\sum f_i x_i$ / $\sum f_i$

54 = 2200 + 50K / 40 + K

54 [40 + K] = 2200 + 50K

2160 + 54K = 2200 + 50K

4K = 40

K = 10

[ii]

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Frequency	4	16	40	76	96	112	120	125

Mode = Z = $L_1 + (F_1 - F_0) / (2F_1 - F_0 - F_2) * h$

= 30 + [36 - 24] / [72 - 24 - 20]

= 30 + (30 / 7)

= 240 / 7

= 34.2

[iii]

Class	4	7	10	15	8	5	3	
Frequency	30	33	35	40	43	45	48	**52**
$f_i x_i$	120	231	350	600	344	225	144	**2014**
$d_i = x_i$ - A [A = 40]	-10	-7	-5	0	3	5	8	
$f_i d_i$	-40	-49	-50	0	24	25	24	**-66**

Direct method:

Mean = $\sum f_i x_i / \sum f_i$

= 2014 / 52

= 38.73

Assumed mean method:

Mean = a + $\sum f_i d_i / \sum f_i$

= 40 + (-66 / 52)

= 38.73

[Alternative Question For Sightless Candidates]

Question 11: Answer any one question: **[5 x 1 = 5]**

(i) Three sides of a triangle are given. Describe the procedure of construction of the circumcircle of the triangle.
(ii) Describe the process of drawing two tangents to a circle from an external point.

Solution:

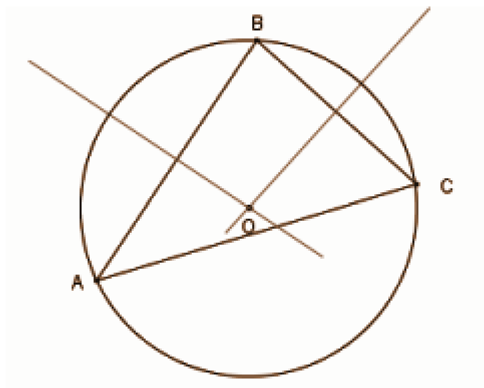
(i) Start with a triangle ABC.

1. Find the bisector of one of the triangle sides.
2. Repeat for another side.
3. Repeat for the third side. three bisectors exist which intersect at a single point.

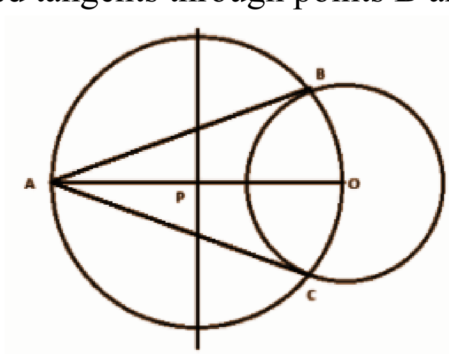
4. The point where these two perpendiculars intersect is the triangle's circumcenter, the centre of the circle we desire.

4. Place the compasses' point on the intersection of the perpendiculars and set the compasses' width to one of the points A, B or C. Draw a circle that will pass through all three.

5. The circle drawn is the triangle's circumcircle, the only circle that will pass through all three of its vertices.



[ii] Step 1: Consider a point A from the outside the circle with centre O.
Step 2: Join points A and O, bisect the line AO. Let P be the midpoint of AO.
Step 3: Draw a circle taking P as centre and PO as a radius. This circle will intersect at two points B and C on the circle with centre O.
Step 4: Join points A with B and C.
AB and AC are the required tangents through points B and C on the circle.



[Additional Question for External Candidates]

Question 16: (a) Answer any three questions: **[2 x 3 = 6]**

(i) p : q = 5 : 7 and p + q = - 4 then what is the value of (3p + 2q)?
(ii) In how many years the interest will be the $(3 / 5)^{th}$ of principal at the rate of 10% simple interest per annum?
(iii) What is the circular value of an angle formed by the endpoint of the hour hand of a clock in 1-hour rotation?
(iv) If x = 2 + √5 and xy = −1, find the value of x − y.

Solution:
[i] p = 5x
q = 7x
p + q = -4
5x + 7x = -4
12x = -4
x = -1 / 3
p = 5 * (-1 / 3) = -5 / 3
q = 7 * (-1 / 3) = -7 / 3
3p + 2q = 3 * (-5 / 3) + 2 * (-7 / 3)
= -15 / 3 - 14 / 3
= -29 / 3

[ii] Interest = (Principal * Rate * Time) / 100
(3 / 5) * P = (P * 10 * T) / 100
3 / 5 = T / 10
T = 30 / 5 = 6 years

[iii] In one complete circle of 12 hours, it completes a 2π angle.
⇒ In 1 hour, it will complete 2π / 12 = π / 6 = 30^{o}

[iv] x = 2 + √5 and xy = −1
y = -1 / x
= −1 / [2 + √5] * (2 − √5 / 2 − √5)
= − (2 − √5) / −1
= 2 − √5
x − y = 2 + √5 − 2 + √5

$= 2\sqrt{5}$

(b) Answer any four questions: **[4 x 1 = 4]**

(i) What is the value of a if one root of the equation $x^2 + ax + 3 = 0$ is 1?
(ii) If the product of three positive continued proportional numbers is 64, what is their mean proportional?
(iii) Find the value of k if the roots of the equation $x^2 - kx + 4 = 0$ are real and equal.
(iv) Find the median of the numbers 1, 2, 3, 5, 8, 6, 9, 11 and 4.
(v) In a partnership business, A invests Rs. 600 for 9 months and B invests Rs. 700 for 5 months, find the ratio of their share of profit.

Solution:
[i] Let α be the one root of the equation and β be the other root.
$x^2 + ax + 3 = 0$, $\alpha = 1$
On comparing the given equation with $ax^2 + bx + c = 0$,
$a = 1$, $b = a$, $c = 3$
$1^2 + a(1) + 3 = 0$
$1 + a + 3 = 0$
$a = -4$

[ii] Let the numbers be x, y, z.
$xyz = 64$
As they are in continued proportion, so $x / y = y / z$
$\Rightarrow y^2 = xz$
$\Rightarrow y^3 = xyz = 64$
$\Rightarrow y = 4$

[iii] $x^2 - kx + 4 = 0$
On comparing the given equation with $ax^2 + bx + c = 0$,
$a = 1$, $b = -k$, $c = 4$
For real and equal roots,
$b^2 - 4ac = 0$
$k^2 - 4 * (1) * 4 = 0$

$k^2 = 16$

$k = \pm 4$

[iv] 1, 2, 3, 5, 8, 6, 9, 11 and 4

On arranging in ascending order, 1, 2, 3, 4, 5, 6, 8, 9, 11.

n = 9

Median = [n + 1] / 2 = [9 + 1] / 2 = 5^{th} term = 5

[v] Investment of A = 600 * 9 = Rs. 5400

Investment of B = 700 * 5 = Rs. 3500

Share of profit = Share of investment

= 5400 * 3500

= 54:35