Madhyamik Question Paper 2018
With Solutions

Question 1: Choose the correct option in each case from the following questions: **[1 x 6 = 6]**

(i) Interest on Rs. a at the simple interest 10% per annum for b months is:

(a) Rs. ab / 100 (b) Rs. ab / 120 (c) Rs. ab / 1200 (d) Rs. ab / 10

Answer: (b)
R = 10%
T = b months = b / 12 years
SI = PTR / 100
= a * b * (10) / 100 * 12
= ab / 120

(ii) If $x \propto y$ then

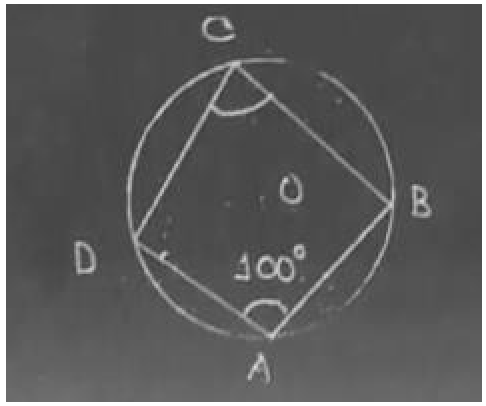
(a) $x^2 \propto y^2$ (b) $x^3 \propto y^2$ (c) $x \propto y^2$ (d) $x^2 \propto y^2$

Answer: (a)
$x \propto y$
$x = ky$
$x^2 = k^2y^2$
$x^2 \propto y^2$

(iii) If $\angle A = 100°$ of a cyclic quadrilateral ABCD, then the value of $\angle C$ is:

(a) 50° (b) 20° (c) 80° (d) 180°

Answer: (c)



$\angle A = 100^\circ$

$\angle A + \angle C = 180^\circ$

$\angle C = 180^\circ - 100$

$\angle C = 80^\circ$

(iv) The sexagesimal value of 7π / 12 is:

(a) 115° (b) 150° (c) 135° (d) 105°

Answer: (d)
$7\pi / 12$
$= (7 * 180) / 12$
$= 105^\circ$

(v) If the side of a cube is a unit and the diagonal of the cube is d unit then the relation between a and d will be.

(a) $\sqrt{2}a = d$ (b) $\sqrt{3}a = d$ (c) $a = \sqrt{3}d$ (d) $a = \sqrt{2}d$

Answer: (b)

(vi) If the mean of the numbers 6, 7, x, 8, y, 16 is 9 then:

(a) $x + y = 21$ (b) $x + y = 17$ (c) $x - y = 21$ (d) $x - y = 9$

Answer: (b)
6, 7, x, 8, y, 16 is 9
Mean = 9
$9 = [6 + 7 + x + 8 + y + 16] / 6$
$54 = 37 + x + y$
$x + y = 17$

Question 2: Fill up the blanks (any five): [1 x 5 = 5]

(i) If the simple interest of a principal for n years at r% p.a. be Rs. pnr / 25, then the principal will be Rs ______. **[4P]**

(ii) The equation $(a - 2) x^2 + 3x + 5 = 0$ will not be a quadratic equation for a = ______. **[a = 2]**

(iii) if ABCD is a cyclic parallelogram then A is ______. **[90º]**

(iv) If tan 35° tan 55°= sin θ, then the lowest positive value of θ will be _______. **[90º]**
$\tan 35° \tan 55° = \sin \theta$
$\tan 35° \tan (90° - 35°) = \sin \theta$
$\tan 35° * \cot 35° = \sin \theta$
$1 = \sin \theta$
$\theta = 90°$

(v) The shape of a pencil with one end sharpened is the combination of a cylinder and a _______ **[cone]**

(vi) The measures of central tendency are Mean, Median and ______. **[Mode]**

Question 3: Write True or False (any five): [1 x 5 = 5]

(i) At the same rate of interest, the simple interest for 2 years is more than the compound interest on the same principal. **[False]**

(ii) x^3y, x^2y^2 and xy^3 are in continued proportion. **[True]**

(iii) The angle in the segment of a circle which is less than a semicircle is an obtuse angle. **[True]**

(iv) Simplest value of $\sec^2 27° - \cot^2 63°$ is 1. **[True]**

$\sec^2 27° - \cot^2 63°$

$= \sec^2 27° - \cot^2 [90 - 27]$

$= \sec^2 27° - \tan^2 27°$

$= 1$

(v) If the radius of a sphere is twice that of the 1st sphere then the volume of the sphere will be twice that of the 1st sphere. **[False]**

(vi)

Score	1	2	3	4	5
Number of students	3	6	4	7	5

The mode of the distribution is 3. **[False]**

Question 4: Answer any one question: **[3 x 1 = 3]**

[i] The rate of simple interest per annum reduces from 4% to 3 (3 /4) % and for this, a person's annual income decreases by Rs. 60. Determine the principal of that person.

Solution:

SI (1) = PTR / 100

= P * 4 * 1 / 100

= P / 25

Total income = P + (P / 25) = 26P / 25

SI (2) = PTR / 100
= P * (15 / 4) * 1 / 100
= 3P / 80
Total income = P + (3P / 80) = 83P / 80
[26P / 25] - 60 = 83P / 80
26P - 1500 / 25 = 83P / 80
415P = 416P - 24000
P = 24000

[ii] A and B start a business with Rs. 15,000 and Rs. 45,000, respectively. After 6 months B received Rs 3,030 as profit. What is A’s profit?

Solution:
Amount invested by A = Rs. 15000
Amount invested by B = Rs. 45000
The ratio of their profits after 6 months would be
A:B
15000:45000
15:45
1:3
Profit earned by B = Rs. 3030
According to the question, it becomes,
3x = 3010
x = 3010 / 3
x = 1010
So, A's profit after 6 months would be Rs. 1010.

[iii] If 2x + [1 / x] = 2, then find the value of x / [2x^2 + x + 1].

Solution:
2x + [1 / x] = 2
2x^2 + 1 = 2x
LHS = x / [2x^2 + x + 1]
= x / 2x^2 + 1 + x
= x / 2x + x

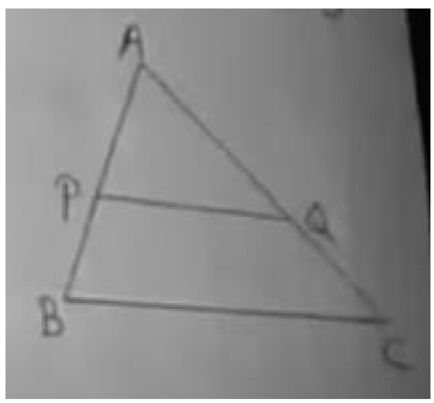
$= x / 3x$
$= 1 / 3$

[iv] If the roots of a quadratic equation are 2 and -3, then write the equation.

Solution:
$a = 2, b = -3$
$x^2 - (a + b) x + ab = 0$
$x^2 - (2 + (-3))x + (2 * -3) = 0$
$x^2 + x - 6 = 0$

[v] The line parallel to BC of △ABC meets AB and AC at P and Q respectively. If AP = 4 cm, QC = 9 cm and PB = AQ, then find the length of PB.

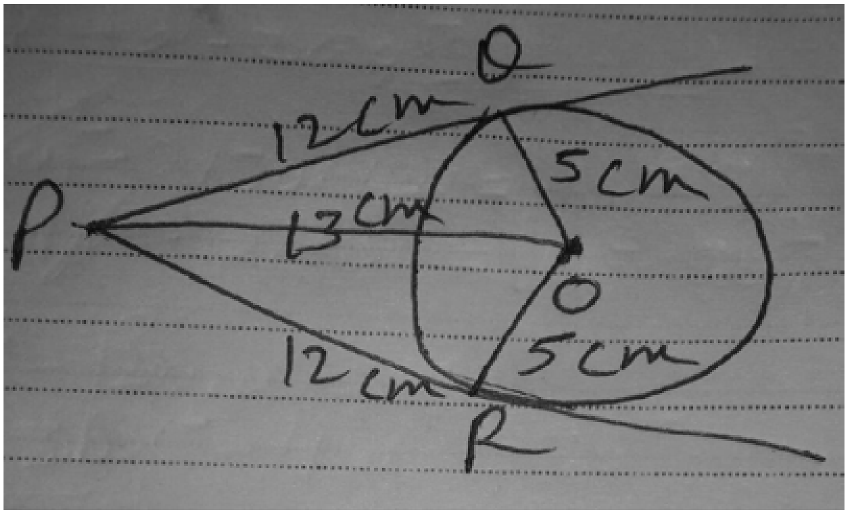
Solution:



Since PQ || BC
By basic proportionality theorem,
AP / PB = AQ / QC
$4 / x = x / 9$
$36 = x^2$
$x = 6cm$
Length of PB is 6cm.

[vi] The radius of a circle with centre O is 5 cm. P is a point at a distance 13 cm from O. PQ and PR are two tangents to this circle. Find the area of the quadrilateral PQOR.

Solution:



PQ & PR are 2 tangents and QO & OR are 2 radii at contact point Q & R.

$\angle$PQO = 90° [a tangent to a circle is perpendicular to the radius through the point of contact]

By Pythagoras theorem

$PQ^2 = OP^2 - OQ^2$

$PQ^2 = 13^2 - 5^2$

$= 169 - 25$

$= 144$

$PQ = \sqrt{144} = 12$

PQ = 12cm

PQ = PR = 12cm [The lengths of two tangents drawn from an external point to a circle are equal]

In ΔOPQ & Δ OPR

OQ = OR (5cm) given

OP = OP (Common)

PQ = PR (12cm)

Hence, ΔOPQ and ΔOPR are congruent. (by SSS congruence)

Area of ΔOPQ = Area ΔOPR

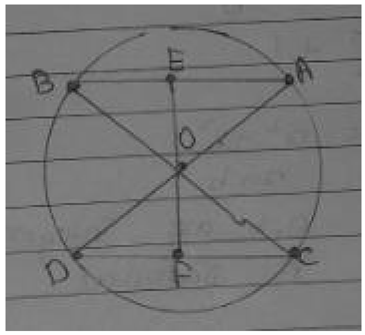
Area of quadrilateral QORP = 2 × (area of Δ OPR)

Area of quadrilateral QORP = 2 × 1 / 2 × base × altitude

Area of quadrilateral QORP = OR × PR

Area of quadrilateral QORP $= 12 \times 5$
$= 60\ cm^2$
[vii] The two chords AB and CD of a circle are at equal distance from the centre O. If $\angle AOB = 60°$ and CD = 6 cm, then calculate the length of the radius of the circle.

Solution:



In △AOB and △COD,
AB = CD
OA = OC = OB = OD
All the angles and sides should be equal.
AB = 6cm
$\angle COD = 60^o$
AE = AB / 2
AE = 6 / 2 = 3cm
In △AOE and △BOE,
OA = OB
OE = OE
AE = BE
By SSS congruence,
△AOE ≅ △BOE
$\angle AOE = \angle BOE$
$\angle AOE = 30^o$
sin θ = perpendicular / hypotenuse
$\sin 30^o = AE / OA$
[1 / 2] = AE / OA
OA = 3 * 2

OA = 6cm

[viii] If $\tan\theta + \cot\theta = 2$, then find the value of $\tan^7\theta + \cot^7\theta$.

Solution:

$\tan\theta + \cot\theta = 2$

$\Rightarrow \tan\theta + 1 / \tan\theta = 2$

$\Rightarrow \tan^2\theta + 1 = 2\tan\theta$

$\Rightarrow \tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow (\tan\theta - 1)^2 = 0$

$\Rightarrow \tan\theta = 1$

$\cot\theta = 1 / \tan\theta = 1$

$\tan^7\theta + \cot^7\theta$

$= (\tan\theta)^7 + (\cot\theta)^7$

$= (1)^7 + (1)^7$

$= 1 + 1$

$= 2$

[ix] If the ratio of the length of the shadow of a tower and height of the tower is $\sqrt{3}:1$, find the angle of elevation of the sun.

Solution:

The ratio of the height of a tower and the length of its shadow is given by $\sqrt{3}:1$.

$\tan\theta$ = Height of tower / Length of shadow

$\tan\theta = \sqrt{3} / 1$

$\tan\theta = \tan 60^\circ$

$\theta = 60^\circ$

Hence, the angle of the elevation of the sun is 60°.

[x] The volumes of two right circular cylinders are the same. The ratio of their height is 1:2. Find the ratio of their radii.

Solution:
The volume of a right circular cylinder with radius r and height h is $V = \pi r^2 h$.
It is given that the ratio of the heights of two circular cylinders is 1:2 that is $h_1 / h_2 = 1 / 2$
$V_1 = V_2$
$\Rightarrow \pi r_1^2 h_1 = \pi r_2^2 h_2$
$\Rightarrow r_1^2 / r_2^2 = h_2 / h_1$
$\Rightarrow r_1^2 / r_2^2 = 1 / [h_1 / h_2]$
$\Rightarrow r_1^2 / r_2^2 = 1 / (1 / 2)$
$\Rightarrow (r_1 / r_2)^2 = 2$
$\Rightarrow r_1 / r_2 = \sqrt{2}$
Hence, the ratio of their radius is $\sqrt{2}:1$.

[xi] The volume of a solid hemisphere is 144 π cubic cm, then find the diameter of the sphere.

Solution:
The volume of the hemisphere = $2\pi r^3 / 3$
$144\pi = 2 * (22 / 7) * r^3 / 3$
$144 * 3 = 2 * r^3$
$216 = r^3$
r = 6cm
d = 2 * r = 2 * 6 = 12cm

[xii] The mean of a frequency distribution is 8.1 if $\sum f_i x_i = 132 + 5K$ and $\sum f_i = 20$ then what is the value of K?

Solution:
Mean = $\sum f_i x_i / \sum f_i$
Mean = 8.1
$\sum f_i x_i = 132 + 5K$
$\sum f_i = 20$

$8.1 = (132 + 5k) / 20$
$8.1 \times 20 = 132 + 5k$
$162 = 132 + 5k$
$162 - 132 = 5k$
$30 = 5k$
$k = 6$

Question 5: Answer any one question : [5 x 1 = 5]

(a) Aminur has taken a loan of Rs. 64, 000 from a bank. If the rate of interest is 2.5 paise per rupee per annum, calculate the compound interest payable after 2 years.
(b) A, B and C start a business with the capital of Rs. 6,000, Rs 8,000 and Rs. 9, 000, respectively. After a few months, A invests Rs 3, 000 more in the business. At the end of the year, they gained Rs 30,000 and C got Rs. 10,800 as a share of profit. When did A invest Rs. 3,000 more?

Solution:
[a] P = Rs. 64000
r = 2.5 paise per rupee per annum (given)
= 0.025 rupee per rupee per annum
= 0.025 x 100 rupee per hundred rupee per annum
= 0.025 x 100 per cent per annum
= 2.5 percent per annum
t = 2 years

C.I. = $64000 [(1 + 2.5 / 100)^2]$
= $64000 [(1.025)^2]$
= 64000 x 1.050625
= 67240
= Rs. 67240
CI = 67240 - 64000 = Rs. 3240

[b] A invests Rs 3, 000 more in the business.
A = 6000 + 3000 = 9000

= 6000 * x + 9000 (12 - x)
= 6000x + 108000 - 9000x
= 108000 - 3000x
= 3000 (36 - x)
B invested Rs. 8000
= (8000 * 12)
= Rs. 96000
C invested Rs. 9000
= (9000 * 12)
= Rs. 108000
Ratio of A, B and C together
= 3000 (36 - x) : 96000 : 108000
= (36 - x) : 32 : 36
Gain = Rs. 30000
C = 30000 * [36 / (36 - x) + 32 + 36]
= 30000 * [36 / 104 - x]
30000 * [36 / 104 - x] = 10800
36 / 104 - x = 10800 / 30000
936 - 9x = 900
-9x = -36
x = 4

Question 6: Answer any one question: **[3 x 1 = 3]**

(a) Solve: $\{[x + 4] / [x - 4]\}^2 - 5 [x + 4 / x - 4] + 6 = 0, (x \neq 4)$
(b) The digit in the unit's place of a two-digit number is 6 more than that at the ten's place. The product of the digits is 12 less than the number. Find the possible values of the digit in the unit place.

Solution:
[a] $\{[x + 4] / [x - 4]\}^2 - 5 [x + 4 / x - 4] + 6 = 0$,
Take [x + 4 / x - 4] = a
$a^2 - 5a + 6 = 0$
$a^2 - 3a - 2a + 6 = 0$
a (a - 3) - 2 (a - 3) = 0

$(a - 3)(a - 2) = 0$

$a = 3, 2$

$a = 3$

$x + 4 / x - 4 = 3$

$x + 4 = 3(x - 4)$

$x + 4 = 3x - 12$

$12 + 4 = 3x - x$

$16 = 2x$

$x = 8$

$a = 2$

$x + 4 / x - 4 = 2$

$x + 4 = 2(x - 4)$

$x + 4 = 2x - 8$

$8 + 4 = 2x - x$

$12 = x$

[b] $x(x + 6) = 10x + x + 6 - 12$

$x^2 + 6x = 11x - 6$

$x^2 - 5x + 6 = 0$

$(x - 3)(x - 2) = 0$

$x = 3, 2$

$3 + 6 = 9$

$2 + 6 = 8$

Question 7: Answer any one question: **[3 x 1 = 3]**

(a) Find the simplest value of $\sqrt{7}(\sqrt{5} - \sqrt{2}) - \sqrt{5}(\sqrt{7} - \sqrt{2}) + 2\sqrt{2} / \sqrt{5} + \sqrt{7}$.

(b) If $x \propto y$ and $y \propto z$, then prove that: $(x^2 + y^2 + z^2) \propto (xy + yz + xz)$

Solution:

[a] $\sqrt{7}(\sqrt{5} - \sqrt{2}) - \sqrt{5}(\sqrt{7} - \sqrt{2}) + 2\sqrt{2} / \sqrt{5} + \sqrt{7}$

$= \sqrt{35} - \sqrt{14} - \sqrt{35} + \sqrt{10} + [2\sqrt{2}(\sqrt{7} - \sqrt{5}) / (\sqrt{5} + \sqrt{7})(\sqrt{7} - \sqrt{5})]$

$= \sqrt{35} - \sqrt{14} - \sqrt{35} + \sqrt{10} + 2\sqrt{2}\ (\sqrt{7} - \sqrt{5}) / (\sqrt{7})^2 - (\sqrt{5})^2$
$= \sqrt{35} - \sqrt{14} - \sqrt{35} + \sqrt{10} + 2\sqrt{2}\ (\sqrt{7} - \sqrt{5}) / 2$
$= \sqrt{35} - \sqrt{14} - \sqrt{35} + \sqrt{10} + \sqrt{14} - \sqrt{10}$
$= 0$

[b] $x \propto y$
$x = k_1y$
$y \propto z$
$y = k_2z$
Hence, $x = k_1k_2z$
$(x^2 + y^2 + z^2) \propto (xy + yz + xz)$
$= (k_1k_2z)^2 + (k_2z)^2 + z^2 / [k_1y * k_2z + k_2z * z + k_1y * z]$
$= z^2\ (k_1^2\,k_2^2 + k_2^2 + 1) / [k_1k_2^2z^2 + k_2z^2 + k_1k_2z^2]$
$= z^2\ (k_1^2\,k_2^2 + k_2^2 + 1) / z^2\ [k_1k_2^2 + k_2 + k_1k_2]$
$= (k_1^2\,k_2^2 + k_2^2 + 1) / [k_1k_2^2 + k_2 + k_1k_2]$
So, $(x^2 + y^2 + z^2) \propto (xy + yz + xz)$

Question 8: Answer any one question: [3 x 1 = 3]

[a] If [a + b − c] / [a + b] = [b + c − a] / [b + c] = [c + a − b] / [c + a] and a + b + c ≠ 0 then prove that a + b = c.
[b] If x:a, y:b, z:c that show that $(a^2 + b^2 + c^2)(x^2 + y^2 + z^2) = (ax + by + cz)^2$.

Solution:
[a] [a + b - c] / [a + b] = [b + c - a] / [b + c] = [c + a - b] / [c + a]
[a + b] / [a + b] - [c] / [a + b] = [b + c] / [b + c] - [a] / [b + c] = [c + a] / [c + a] - [b] / [c + a]
1 - [c] / [a + b] = 1 - [a] / [b + c] = 1 - [b] / [c + a]
[c] / [a + b] = [a] / [b + c] = [b] / [c + a]
a + b / c = b + c / a = c + a / b
a + b + c / c = b + c + a / a = c + a + b / b
1 / c = 1 / a = 1 / b
c = a = b or a = b = c

[b] x:a, y:b, z:c

x / a = y / b = z / c = k [Say]

Let x = ka, y = kb, z = kc

LHS = $(a^2 + b^2 + c^2)(x^2 + y^2 + z^2)$

$= (a^2 + b^2 + c^2)(k^2a^2 + k^2b^2 + k^2c^2)$

$= k^2 (a^2 + b^2 + c^2)(a^2 + b^2 + c^2)$

$= k^2 (a^2 + b^2 + c^2)^2$

RHS = $(ax + by + cz)^2$

$= (a * ka + b * kb + c * kc)^2$

$= (k^2a^2 + k^2b^2 + k^2c^2)^2$

$= k^2 (a^2 + b^2 + c^2)^2$

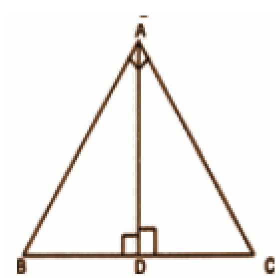
Question 9: Answer any one question: **[5 x 1 = 5]**

(a) Prove that, if a perpendicular is drawn on the hypotenuse from the right angular point of a right-angled triangle, two triangles so formed on the two sides of the perpendicular are each similar to the original triangle and also similar to each other.

(b) Prove that the tangent and the radius through the point of contact of a circle are perpendicular to each other.

Solution:

[a]



Given a right angle triangle, right-angled at A.

AD is the perpendicular drawn to the hypotenuse BC from vertex A.

To Prove:

(i) $\Delta BDA \sim \Delta BAC$

(ii) $\Delta ADC \sim \Delta BAC$

(iii) $\Delta BDA \sim AADC$

Proof:

In ΔBDA and ΔBAC:

$\angle ADB = \angle A = 90°$

$\angle B = \angle B$ [common]

Therefore, by using AA similar condition,

$\Delta BDA \sim \Delta BAC$ ...(i)

Now, in ΔADC and ΔBAC,

$\angle ADC = \angle A = 90°$

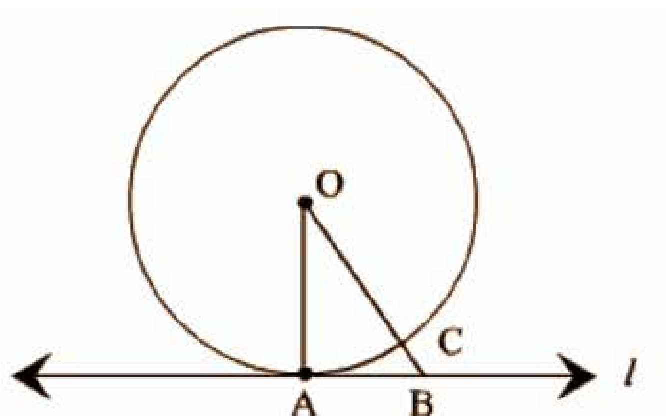
$\angle C = \angle C$ [common]

Therefore, by using AA similar condition,

$\Delta ADC \sim \Delta BAC$ ...(ii)

Comparing (i) and (ii), $\Delta BDA \sim \Delta ADC$.

[b]



Given: A circle C (0, r) and a tangent l at point A.

To prove: $OA \perp l$

Construction: Take a point B, other than A, on the tangent l. Join OB. Suppose OB meets the circle in C.

Proof: We know that, among all line segments joining the point O to a point on l, the perpendicular is shortest to l.

OA = OC (Radius of the same circle)

Now, OB = OC + BC.

$\therefore OB > OC$

$\Rightarrow OB > OA$

$\Rightarrow OA < OB$

B is an arbitrary point on the tangent l. Thus, OA is shorter than any other line segment joining O to any point on l.

Here, $OA \perp l$.

Question 10: Answer any one question: [3 x 1 = 3]

(a) In triangle ABC, AD is perpendicular on BC and $AD^2 = BD. DC$, prove that $\angle BAC$ is a right angle.

(b) A straight line intersects one of the two concentric circles at the points A and B and another at the points C and D. Prove that AC = BD.

Solution:

[a] Given: In triangle ABC, AD is perpendicular to BC and $AD^2 = BD.DC$

To prove: $\angle BAC = 90°$

Proof: In right triangles ΔADB and ΔADC, Pythagoras theorem should be applied,

$AB^2 = AD^2 + BD^2$ ---------- (1)

$AC^2 = AD^2 + DC^2$ --------- (2)

$AB^2 + AC^2 = 2AD^2 + BD^2 + DC^2$

$= 2BD . CD + BD^2 + CD^2$ [$\because$ given $AD^2 = BD.CD$]

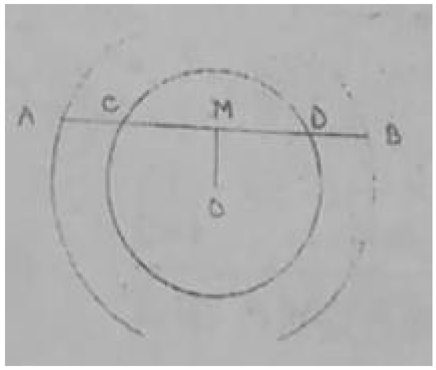
$= (BD + CD)^2 = BC^2$

Thus in triangle ABC, $AB^2 + AC^2 = BC^2$

Hence triangle ABC is a right triangle right angled at A.

$\angle BAC = 90°$

[b]

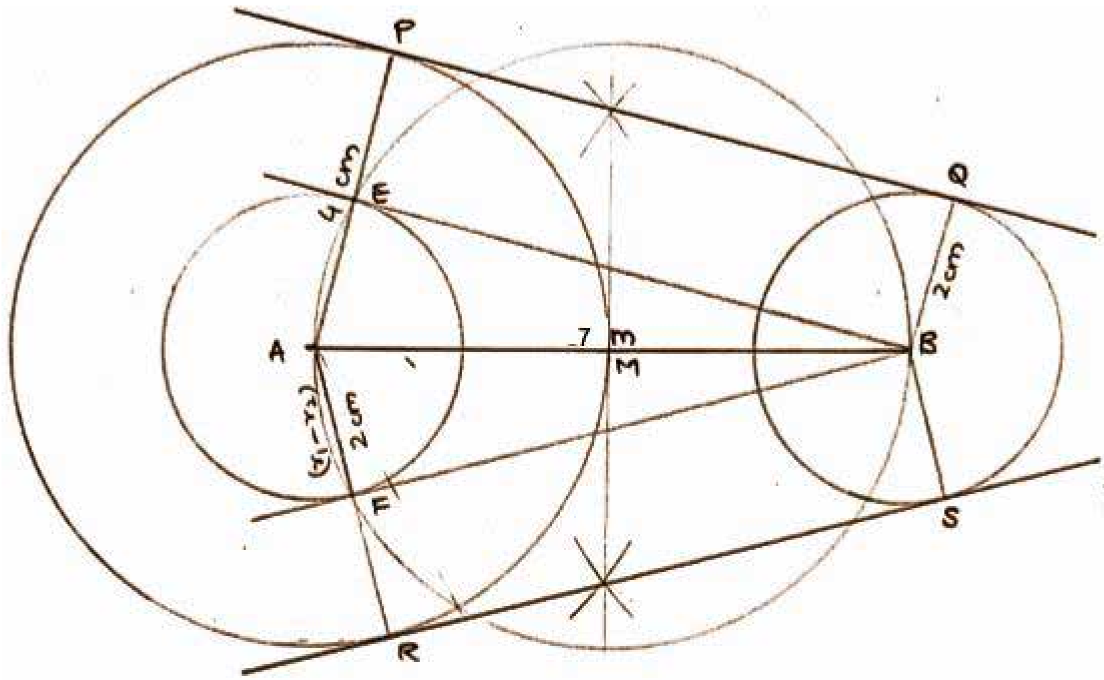


Given: O is the centre and a straight line intersects one of the two concentric circles at the points A and B and other at the points C and D.
To prove AC = BD
Construction: OM is drawn perpendicular to AB
Proof:
CM = DM [perpendicular drawn from the centre of the circle to the chord bisects the chord]
AM - CM = BM - DM
AC = BD

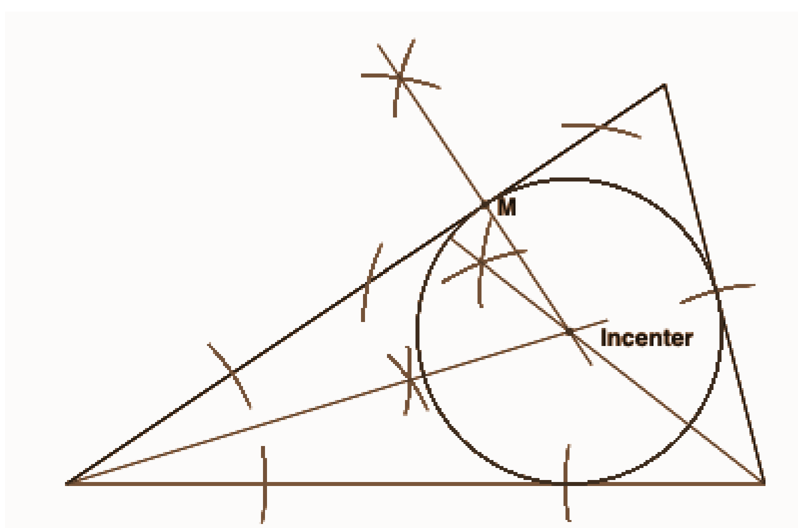
Question 11: Answer any one question: **[5 x 1 = 5]**

(a) Constant two circles of radii 4 cm and 2 cm and the distance between their centres is 7 cm. Construct a direct common tangent of the circles. (only traces of construction are required).
(b) Construct a triangle whose two sides are 9 cm and 7 cm and the angle between them is 60°. Construct the incircle of the triangle. (only traces of construction are required).

Solution:
[a]



[b]



Question 12: Answer any two questions: **[3 x 2 = 6]**

(a) An arc of length 220 cm of a circle makes an angle 60° at the centre. Find the radius of the circle.

(b) If $\cos^2 \theta - \sin^2 \theta = 1 / 2$, then find the value of $\tan^2 \theta$.

(c) Find the value of $\sec 17^o / \operatorname{cosec} 73^o + \tan 68^o / \cot 22^o + \cos^2 44 + \cos^2 46^o$.

Solution:

[a] Arc length = θ / 360 * $2\pi r$

Given that an arc of length 220cm of a circle makes an angle 60° at the centre,

220 = {60} / {360} * 2 * {22} / {7} r

220 = (1 / 6) * (44 / 7) * r

220 / 1.048 = r

210 = r

Hence the radius of the circle is 210 cm.

[b] $\cos^2\theta - \sin^2\theta = 1 / 2$

$[\cos^2\theta - \sin^2\theta + 1] / [\cos^2\theta - \sin^2\theta - \sin^2\theta - \cos^2\theta] = [1 + 2] / [1 - 2]$

$[\cos^2\theta - \sin^2\theta + \sin^2\theta + \cos^2\theta] / [\cos^2\theta - \sin^2\theta - \sin^2\theta - \cos^2\theta] = 3 / -1$

$2\cos^2\theta / - 2\sin^2\theta = 3 / -1$

$\sin^2\theta / \cos^2\theta = 1 / 3$

$\tan^2\theta = 1 / 3$

[c] $\sec 17^o / \text{cosec } 73^o + \tan 68^o / \cot 22^o + \cos^2 44 + \cos^2 46^o$

$= [\sec 17^o / \text{cosec } (90^o – 73^o)] + [(\tan 90^o – 22^o) / \cot 22^o] + \cos^2 (90^o – 44^o) + \cos^2 46^o$

$= [\sec 17^o / \sec 17^o] + [\cot 22^o / \cot 22^o] + [\sin^2 46^o + \cos^2 46^o]$

= 1 + 1 + 1

= 3

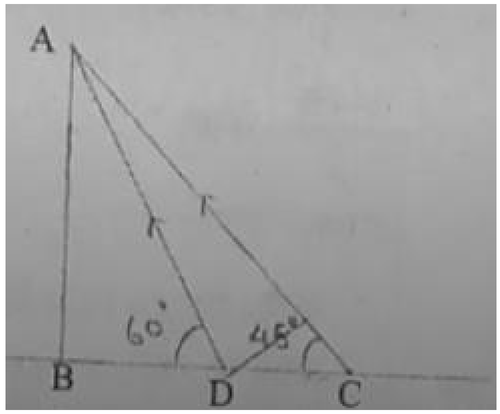
Question 13: Answer any one question: [5 x 1 = 5]

(a) The length of the shadow of a post becomes 3 meters smaller when the angle of elevation of the Sun increases from 45° to 60°. Find the height of the post.

(b) A man standing on a railway bridge 5√3 meters high, observes the engine of a train at an angle of depression 30°. But after 2 seconds, he observes the engine at an angle of depression 45° on the other side of the bridge. Find the speed of the train.

Solution:

[a]



$\angle BCA = 45^o$

$\angle BDA = 60^o$

CD = 3m

To find BD

In $\triangle ABD$,

AB / BD = tan 60^o

$AB / x = 1 / \sqrt{3}$

$AB = x * \sqrt{3}m$

In $\triangle ABC$,

AB / BC = tan 45^o

$x\sqrt{3} / x + 3 = 1$

$x\sqrt{3} = x + 3$

$x\sqrt{3} - x = 3$

$x(\sqrt{3} - 1) = 3$

$x = 3 / (\sqrt{3} - 1)$

By rationalising the denominator,

$x = 3\sqrt{3} + 3 / 2$

$AB = \sqrt{3}(3\sqrt{3} + 3 / 2)$

$AB = 9 + 3\sqrt{3} / 2$

= 9 + 5.193 / 2

= 7.098m

[b] Height(h) of the bridge = AB = $5\sqrt{3}$ m

The angle of depression from one side = $30^\circ = \angle ACB$

The angle of depression from other side = $45^\circ = \angle ADB$

Required time (t) = 2 seconds

Speed of the train = ?

Now, from the ΔADB,

$\tan 45^\circ = AB / BD$

$1 = 5\sqrt{3} / BD$

$BD = 5\sqrt{3}$m

Now, from the ΔACB,

$\tan 30^\circ = AB / BC$

$1 / \sqrt{3} = 5\sqrt{3} / BC$

BC = 15m

CD = BC + BD

$CD = 15 + 5\sqrt{3}$ m

Therefore, the distance covered by the train in 2 seconds is

$= (15 + 5\sqrt{3})$ m

Speed of the train = $(15 + 5\sqrt{3})) / 2 = 11.83$ m / s

Question 14: Answer any TWO questions: [4 x 2 = 8]

(a) Each side of a cube is decreased by 50%. Calculate the ratio of the volumes of the original and changed cube.

(b) The total surface area of a right circular cylindrical pot without a lid be 200 sq.cm. If the radius of the base is 7 cm find the quantity of water in litres contained in the pot. (1 litre = 1 cubic dm)

(c) A tank of length 21 dcm, breadth 11 dcm and 6 dcm deep is half-filled with water. If 100 solid iron balls of diameter 21 cm are completely immersed in the tank, then how much dcm of water level is raised?

Solution:

[a] Let length of the cube be x unit

$V = (\text{Side})^3$

$V = (x)^3$ unit^3

Now, when the length of cube is reduced by 50%

New length = x - x * 50 / 100

= x - [x / 2]

= (2x - x) / 2

= x / 2 unit

New volume = $(\text{side})^3$

= $(x / 8)^3$ unit^3

Ratio = Original cube volume:New cube volume

= $x^3 / (x^3 / 8)$

= 8 : 1

[b] $2\pi rh = \pi r^2$

$\pi r(2h + r) = 2002$

(22 / 7) * 7(2h + 7) = 2002

2h + 7 = 91

2h = 84

h = 42

$V = \pi r^2 h$

= (22 / 7) * 7^2 * (42)

= 22 * 7 * 42

= 6.468

= 6.468 / 1000

= 6.468 dcm

[c] Length = 21 dcm

Breadth = 11 dcm

6 dcm deep

Water level raised = x dcm

Volume of the tank = (21 * 11 * x) cm^3

d = 21 / 2 cm

= 21 / 20 d cm

100 iron balls immersed in the tank.

= 100 * (4 / 3) * (22 / 7) * $(21 / 20)^3$ dcm^3

(21 * 11 * x) = 100 * (4 / 3) * (22 / 7) * $(21 / 20)^3$

$231x = 485.1$
$x = 231/\ 485.1$
$x = 2.1$ dcm

Question 15: Answer any two questions: **[4 x 2 = 8]**

(a) Find the mode from the following frequency distribution table of ages of examinees of an entrance examination:

Age (in years)	**16 - 18**	**18 - 20**	**20 - 22**	**22 - 24**	**24 - 26**
Number of examinees	**45**	**75**	**38**	**22**	**20**

(b) Find the median of the given data:

Class Interval	**1 - 5**	**6 - 10**	**11 - 15**	**16 - 20**	**21 - 25**	**26 - 30**	**31 - 35**
Frequency	**2**	**3**	**6**	**7**	**5**	**4**	**3**

(c) From the frequency distribution table given below, draw less than ogive:

Marks obtained	**50 - 60**	**60 - 70**	**70 - 80**	**80 - 90**	**90 - 100**
Frequency	**4**	**8**	**12**	**6**	**10**

Solution:
[a] The most frequently occurring frequency = 75
The modal class = 18 - 20
Mode = $Z = L_1 + (F_1 - F_0) / (2F_1 - F_0 - F_2) * i$
$= 18 + [75 - 45] / [2 * 75 - 45 - 38] * 2$

= 18 + [30 / 67] * 2
= 18.9

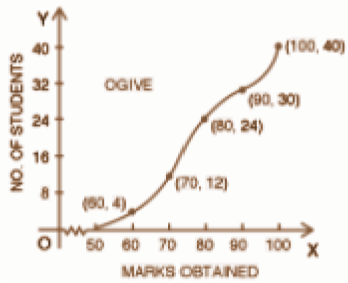
[b]

Class Interval	1 - 5	6 - 10	11 - 15	16 - 20	21 - 25	26 - 30	31 - 35
Class Interval	0.5 - 5.5	5.5 - 10.5	10.5 - 15.5	15.5 - 20.5	20.5 - 25.5	25.5 - 30.5	30.5 - 35.5
Frequency	2	3	6	7	5	4	3
CF	2	5	11	18	23	27	30

n = 30
Median = n / 2 = 30 / 2 = 15
Median class = 15.5 - 20.5
Median = m = l + [(n / 2 – CF) / f] * h
= 15.5 + [(30 / 2) - 11] / 7] * 5
= 15.5 + 2.86
= 18.36

[c]

Marks obtained	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100
Frequency	4	8	12	6	10
CF	4	12	24	30	40


Y
40
32
24
16
8
O
NO. OF STUDENTS
OGIVE
(100, 40)
(90, 30)
(80, 24)
(70, 12)
(60, 4)
50
60
70
80
90
100
X
MARKS OBTAINED