Madhyamik Maths Question Paper 2016 With Solutions

PART - I

Question 1: Answer all questions: **[1 x 6 = 6]**

[i] At simple rate interest of Rs 9,999 amounts to its double in 10 yrs. What is the rate of interest?

Solution:
The amount invested is 9999.
Double the amount means: $9999 \times 2 = 19998$
S = PTR / 100
9999 = 9999 * 10 * R / 100
9999 * 100 = 9999 * 10 * R
999900 / 99990 = R
R = 10%

[ii]: What is the coefficient of x° in polynomial $2x^3 - 3x^2 + 4x + 5$?

Solution: 5

[iii]: What is the H.C.F of $4p^2qr^3$ and $6p^3q^2r^4$?

Solution:
Factor of $4p^2qr^3$
$= 4p^2qr^3$
$= 2 \times 2p^2qr^3$
$= 2 \times 2\ p^2qr^3$
Factor of $6p^3q^2r^4$
$= 6p^3q^2r^4$
$= 2 \times 3p^3q^2r^4$

$= 2 \times 3\ p^3q^2r^4$
HCF = $2p^2qr^3$

[iv]: Find the mixed ratio of a:bc, b:ca and c:ab.

Solution:
a / bc : b / ca : c /ab
For ratio a : bc, b : ca and c : ab; the mixed ratio is (a × b × c) : (bc × ca × ab).

[v]: ABCD is a cyclic trapezium and AD || BC. If ∠ABC = 70°, find the value ∠BCD.

Solution:
In cyclic trapezium,
∠A = ∠D and ∠B = ∠C
∴ ∠BCD = ∠ABC = 70°

[vi] Which one in the following is correct? If tan θ = 0 then:
(a) sin θ = 0 (b) cos θ = 0 (c) cot θ = 0 (d) None of these

Solution: (a)

Question 2: Answer all questions: [2 x 7 = 14]

(i) The average of the first 21 natural numbers is 11. What will be the average of the first 20 natural numbers?

Solution:
N (N + 1) ÷ 2
= 20 (20+1) ÷ 2
= 20 * (21) ÷ 2
= 210

Average = 210 ÷ 20

= 10.5

(ii) If xy > 0, then in which quadrant (x, y) will lie?

Solution:
The first and third quadrant.

(iii) If x is a perfect square number and 0 < ([x - 3] / 2) < 1, find the value of x.

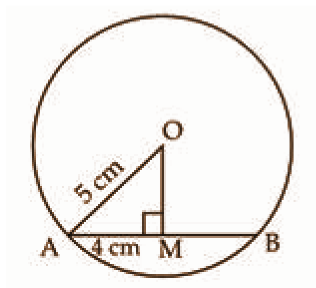
Solution:
0 < ([x - 3] / 2)
([x - 3] / 2) < 1
0 < x - 3
x - 3 < 2
x = 3, 5

(iv): In △ABC, the straight line parallel to the side BC meets AB and AC at the point D and E, respectively. If AE = 2AD ; find DB : EC.

Solution:
AE = 2AD
AE : AD = 2 / 1
By applying BPT
AD / AB = DE / BC = AE / AC
AD / AB = AE / AC
⇒ AB/AD = AC/AE
⇒ (AB / AD) - 1 = (AC / AE) - 1
⇒ (AB - AD) / AD = (AC - AE) / AE
⇒ BD / AD = CE / AE
⇒ AD / BD = AE / CE
⇒ AD / AE = BD / CE
⇒ AD / 2AD = BD / CE
⇒ 1 / 2 = BD / CE
DB : EC = 1: 2

[v] The length of the radius of a circle with centre O is 5 cm and the length of the chord AB is 8 cm. What is the distance of the chord AB from point O?

Solution:



Radius of circle (r) = 5 cm
AB = 8 cm
OM AB
AM = MB = 4 cm [drawn from the centre of the circle bisects the chord]
In right OMA, by Pythagoras theorem
$OA^2 = OM^2 + AM^2$
$5^2 = OM^2 + (4)^2$
$25 = OM^2 + 16$
OM2 = 9
OM = 3 cm

[vi] The volume and area of the base of a right pyramid are 40 cubic cm and 24 sq. cm, respectively. Find its height.

Solution:
Volume of the pyramid = (1 / 2) * base area * height
40 = (1 / 2) * (24) * h
80 / 24 = h
h = 3.33 cm

[vii] From the result, tan (90º - θ) = cot θ, (0 < θ < 90º), show that cot (90º - θ) = tan θ.

Solution:

$\cot(90° - \theta) = 1 / \tan(90° - \theta)$

$\cot(90° - \theta) = 1 / \cot\theta$

$\cot(90° - \theta) = \tan\theta$

PART - II

Question 3: Answer any TWO questions (algebraic method may be applied):
[5 x 2 = 10]

(a) A man borrowed a sum of money at 5% compound interest per annum. He repays Rs. 3,150 at the end of the first year and Rs 4,410 at the end of the second year so as to clear the entire loan. How many rupees did he borrow?

Solution:

Rate of Interest is 5 %.

After the first year, he returns Rs. 3150

So, 21 units = 3150

1 unit = 150

20 unit = 150 × 20 = Rs. 3000

Same as for 2nd year

441 units = 4410

1 unit = 10

40 units = 10 × 400 = Rs.4000

Total sum is = 3000 + 4000 = Rs. 7000

(b) Milk contains 89% of water. If a sample of milk is found to contain 90% of water; what is the amount of excess water in 22 litres of such a sample of milk?

Solution:

A sample of 22 litres containing 90% water will have 10% milk that is 2.2-litre milk.

2.2-litre milk in a mixture of 11% milk means the total mixture is 100 × 2.2 / 11 = 20 litre.

So, 2-litre water should be added in a 20-litre mixture of 89% water to make it a 22-litre mixture having 90% water.

(c) In the mobile phone business of a reputed company, the manufacturer, wholesaler and retailer each of them gain 20%. if a customer purchases a mobile phone of that company for Rs. 8,640, find its manufacturing cost.

Solution:
Let cost of manufacturer = Rs.100
Selling price of manufacturer to whole-saler = 100 + 20 = Rs.120
Selling price of whole-saler to retailer = 120 + [120 × (20 / 100)] = Rs. 144
Selling price of retailer to customer = Rs. 144 + [144 × (20 / 100)] = Rs. 172.8
Hence from manufacturer to the customer, the increase in cost 72.8%
If the buying price of the customer is Rs. 8640, then the cost of manufacturer = 8640 × (100 / 172.8) = Rs.5000

(d) The value of a machine in a factory depreciates at the rate of 10% of its value at the beginning of the year. If its value becomes Rs. 43,740 after 3 years, what is its present value?

Solution:
Depreciation rate = 10%
Value after 3 years = Rs. 43740
$A = P [1 - (r / 100)]^n$
$43740 = P [1 - (10 / 100)]^3$
$43740 = P [0.729]$
$43740 / 0.729 = P$
P = Rs. 60000

Question 4: Answer any ONE question: **[4 x 1 = 4]**

(a) Find the H.C.F of $x^2 - y^2$, $x^3 - y^3$, $3x^2 - 5xy + 2y^2$
(b) Find the L.C.M of $ab^4 - 8ab$, $a^2b^4 + 8a^2b$, $ab^4 - 4ab^2$

Solution:

[a] Factor of $\mathbf{x^2 - y^2}$
Rewrite $x^2 = (x)^2$ and $y^2 = (y)^2 = (x)^2 - (y)^2$
$a^2 - b^2 = (a - b)(a + b)$ where $a = x$ and $b = y$
$= (x - y)(x + y)$
Factor of $x^3 - y^3$
Rewrite $x^3 = (x)^3$ and $y^3 = (y)^3 = (x)^3 - (y)^3$
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ where $a = x$ and $b = y$
$= (x - y)(x^2 + xy + y^2)$
$= (x - y)(x^2 + xy + y^2)$
Factor of $3x^2 - 5xy + 2y^2$
The coefficients are $a = 3, b = -5, c = 2$
Find a pair of factors whose product is $a \cdot c = 3 \cdot 2 = 6$ and sum is $b = -5$.
The pairs of factors for 6 and their sums are
$1, 6 : 1 + 6 = 7$
$2, 3 : 2 + 3 = 5$
Factors are $-2, -3$ whose product is $-2 \cdot -3 = 6$ and sum is $(-2) + (-3) = -5$
So use -2 and -3 to split the -5 coefficient on the middle term
$= 3x^2 - 2xy - 3xy + 2y^2$
$= x(3x-2y) - y(3x-2y)$
$= (3x - 2y)(x - y)$
HCF $= (x - y)$

[b] Factor of $ab^4 - 8ab$
$= ab(b^3 - 8)$
Rewrite $b^3 = (b)^3$ and $8 = (2)^3 = ab((b)^3 - (2)^3)$
Here both terms are perfect cubes, so factor using the difference of cubes formula,
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ where $a = b$ and $b = 2$
$= ab(b - 2)(b^2 + 2b + 4)$
$= ab(b - 2)(b^2 + 2b + 4)$
Factor of $a^2b^4 + 8a^2b$
$= a^2b(b^3 + 8)$
Rewrite $b^3 = (b)^3$ and $8 = (2)^3 = a^2b((b)^3 + (2)^3)$

Here both terms are perfect cubes, so factor using the sum of cubes formula,
$a^3 + b^3 = (a + b)(a2 - ab + b2)$ where a = b and b = 2
$= a^2b(b + 2)(b^2 - 2b + 4)$
$= a^2b(b + 2)(b^2 - 2b + 4)$
The factor of $ab^4 - 4ab^2 = ab^4 - 4ab^2 = ab^2(b^2 - 4)$
Rewrite $b^2 = (b)^2$ and $4 = (2)^2$
$= ab^2((b)^2 - (2)^2)$
Here both terms are perfect squares, so factor using the difference of squares formula, $a^2 - b^2 = (a - b)(a + b)$ where a = b and b = 2
$= ab^2(b - 2)(b + 2)$
$= ab^2(b - 2)(b + 2)$
LCM $= a^2b^2(b - 2)(b^2 + 2b + 4)(b + 2)(b^2 - 2b + 4)$

Question 5: Solve (any ONE): [3 x 1 = 3]

(a) (1 / x) + (5 / y) = (21 / 4); (x + y) / (x - y) = (5 / 3)
(b) $(2x + 1)^2 + (x + 1)^2 = 6x + 47$

Solution:
[a]

$$\begin{bmatrix} \left(\frac{1}{x}\right) + \left(\frac{5}{y}\right) = \left(\frac{21}{4}\right) \\ \frac{(x+y)}{(x-y)} = \left(\frac{5}{3}\right) \end{bmatrix}$$

$$\left(\frac{1}{x}\right) + \left(\frac{5}{y}\right) = \left(\frac{21}{4}\right)$$

$$\frac{1}{x}x + \frac{5}{y}x = \frac{21}{4}x$$

$$\frac{5x}{y} = \frac{21}{4}x - 1$$

$$x = -\frac{4y}{20-21y}$$

$$\left[\frac{-\frac{4y}{20-21y}+y}{-\frac{4y}{20-21y}-y} = \frac{5}{3}\right]$$

$$\left[\frac{16-21y}{3(7y-8)} = \frac{5}{3}\right]$$

$$x = -\frac{4\cdot 1}{20-21\cdot 1}$$

$$x = 4$$

$$x = 4,\ y = 1$$

[b] $(2x + 1)^2 + (x + 1)^2 = 6x + 47$

$\Rightarrow 4x^2 + 1 + 4x + x^2 + 1 + 2x = 6x + 47$
$\Rightarrow 5x^2 + 6x + 2 = 6x + 47$
Subtract 6x from both sides
$\Rightarrow 5x^2 + 2 = 47$
$\Rightarrow 5x^2 = 47 - 2$
$\Rightarrow 5x^2 = 45$
$\Rightarrow x^2 = 9$
$\Rightarrow x = \sqrt{9}$
Therefore, the value of x is + 3, -3.

Question 6: Answer any ONE question: **[4 x 1 = 4]**

(a) Two numbers are such that one is less than the other by 3 and their product is 70. Find the numbers.
(b) Length and breadth of the rectangular plot are 5 m more and 3 m less, respectively than the length of the side of a square plot. Area of the rectangular plot is less than twice the area of the square plot by 78 sq. m. Find the length of the side of the square plot.

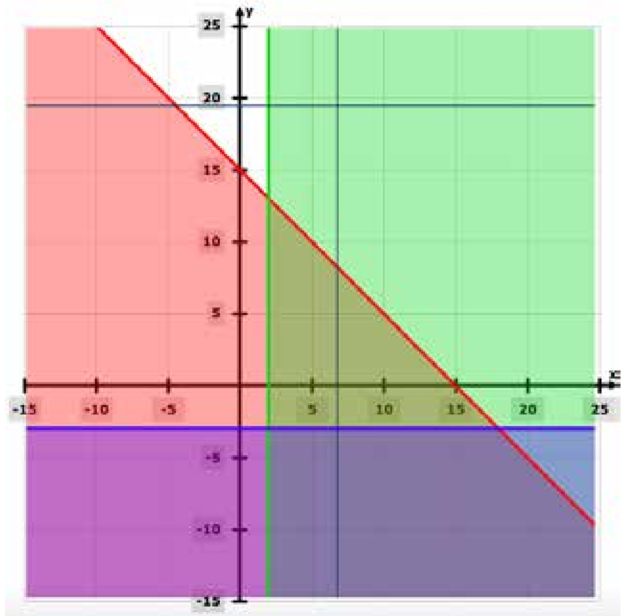
Solution:
[a] Let the two numbers be x and y.
$x = y - 3$ --- (1)
$xy = 70$
$y = 70 / x$ --- (2)
$x = (70 / x) - 3$
$x^2 = 70 - 3x$
$x^2 + 3x - 70 = 0$
$x^2 + 10x - 7x - 70 = 0$
$x (x + 10) - 7 (x + 10) = 0$
$(x - 7) (x + 10) = 0$
$x = 7, -10$
If $x = 7$, then $y = 10$.
If $x = -10$, then $y = -7$.
Therefore, two sets of integers are possible: (7, 10) and (-10, -7).

[b] $A = L * W$
Length of a side of a square plot = a
Length of rectangular plot = a + 5
Breadth of rectangular plot = a - 3
Area of the rectangular plot (R) = 2 * area of the square plot - 78
Find a.
$L * W = 2 * a^2 - 78$
$(a + 5)(a - 3) = 2a^2 - 78$
$a^2 - 3a + 5a - 15 = 2a^2 - 78$
$a^2 + 2a - 15 = 2a^2 - 78$
$a^2 - 2a - 63 = 0$
$a^2 - 9a + 7a - 63 = 0$
$a(a - 9) + 7(a - 9) = 0$
$a = 9, -7$
So the length of the side of the square plot = 9m.

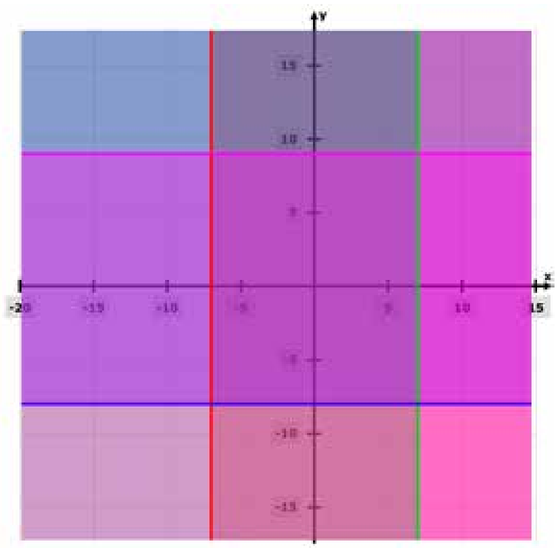
Question 7: Draw the graphs of the following inequalities and indicate the solution region (any ONE): [4 x 1 = 4]

(a) $x + y \leq 15$; $x \geq 2$; $y \leq -3$
(b) $x \geq -7$; $x \leq 7$; $y \geq -8$; $y \leq 9$

Solution:
(a)



(b)



Question 8: Answer any ONE question: **[3 x 1 = 3]**

[a] If $x / y = (a + 2) / (a - 2)$, then show that $(x^2 - y^2) / (x^2 + y^2) = 4a / a^2 + 4$.
[b] If $bcx = cay = abz$, then prove that $(ax + by) / (a^2 + b^2) = (by + cz) / (b^2 + c^2)$.

Solution:
[a] $x / y = (a + 2) / (a - 2)$

$(x^2 / y^2) = (a^2 + 4a + 4) / (a^2 - 4a + 4)$
Add 1 on both sides
$(x^2 / y^2) + 1 = [(a^2 + 4a + 4) / (a^2 - 4a + 4)] + 1$
$(x^2 + y^2) / y^2 = 2a^2 + 8 / (a^2 - 4a + 4)$ ---- (1)
Similarly
$(x^2 - y^2) / y^2 = 8a / (a^2 - 4a + 4)$ ---- (2)
Divide (2) and (1)
$(x^2 - y^2) / (x^2 + y^2) = 8a / 2a^2 + 8$
$(x^2 - y^2) / (x^2 + y^2) = 4a / a^2 + 4$

[b] Since a, b, c are not zero, the given constraint can be rewritten:
$bcx = acy \Rightarrow bx = ay$
$acy = abz \Rightarrow cy = bz$
$abz = bcx \Rightarrow az = cx$
Therefore
$ab (bx - ay) + bc (cy - bz) + ca (cx - az) = 0$
$\Rightarrow ab^2x + bc^2y + ac^2x = a^2by + b^2cz + a^2cz$
$\Rightarrow ab^2x + bc^2y + ac^2x + b^3y = a^2by + b^2cz + a^2cz + b^3y$
$\Rightarrow (ax + by) (b^2 + c^2) = (by + cz) (a^2 + b^2)$
$\Rightarrow (ax + by) / (a^2 + b^2) = (by + cz) / (b^2 + c^2)$

Question 9: Answer any ONE question: [3 x 1 = 3]

[a] If $(x^3 - (1 / y^3)) \propto (x^3 + (1 / y^3))$ then show that $x \propto (1 / y)$.
[b] Volume of a cone is in joint variation with its square of the radius of the base and heights. The ratio of radii of the base of two cones is 3:4 and the ratio of their height is 6:5. Find the ratio of their volumes.

Solution:
[a] $(x^3 - (1 / y^3)) \propto (x^3 + (1 / y^3))$
$[(x^3y^3 - 1) / y^3] / [(x^3y^3 + 1) / y^3] = k$
$(x^3y^3 - 1) / (x^3y^3 + 1) = k$
Using componendo and dividendo,
$(x^3y^3 - 1 + x^3y^3 + 1) / (x^3y^3 - 1 - x^3y^3 - 1) = (k + 1) / (k - 1)$

$2x^3y^3 / (-2) = (k + 1) / (k - 1)$
$(-x^3y^3) = (k + 1) / (k - 1)$
$x^3y^3 = (k + 1) / (1 - k)$
$x^3y^3 = k$ (say some constant)
$x^3 = k / y^3$
So, $x \propto 1 / y$

[b] Let there be cone 1 and cone 2 respectively.
Let the r and R be the radii of the two right circular cones respectively.
Ratio of base radii = 3:5
Ratio of their heights = 6:5
The volume of cone = $1 / 3\ \pi r^2 h$
⇒ The volume of cone 1 / Volume of cone 2
⇒ $(1 / 3 * \pi r^2 h) / (1 / 3\ \pi R^2 h)$
⇒ $(1 / 3 * \pi * 3^2 * 6) / (1 / 3 * \pi * 5^2 * 5)$
$= 3^2 * 6 / 5^2 * 5$
$= 54 / 125$
So, the ratio of their volumes is 54:125.

Question 10: Answer any ONE question: [3 x 1 = 3]

[a] Simplify: $3\sqrt{7} / (\sqrt{5} + \sqrt{2}) - 5\sqrt{5} / (\sqrt{2} + \sqrt{7}) + 2\sqrt{2} / (\sqrt{7} + \sqrt{5})$
[b] If $x = 2 + \sqrt{3}$, $y = 2 - \sqrt{3}$, then find the value of $xy + 1 / xy$.

Solution:
[a]

$$\frac{3\sqrt{7}}{\sqrt{5}+\sqrt{2}} - \frac{5\sqrt{5}}{\sqrt{2}+\sqrt{7}} + \frac{2\sqrt{2}}{\sqrt{7}+\sqrt{5}}$$
$$= \frac{3\sqrt{7}\left(\sqrt{14}+\sqrt{10}+7+\sqrt{35}\right)}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{2}+\sqrt{7}\right)\left(\sqrt{7}+\sqrt{5}\right)} - \frac{5\sqrt{5}\left(\sqrt{35}+5+\sqrt{14}+\sqrt{10}\right)}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{2}+\sqrt{7}\right)\left(\sqrt{7}+\sqrt{5}\right)} + \frac{2\sqrt{2}\left(\sqrt{10}+\sqrt{35}+2+\sqrt{14}\right)}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{2}+\sqrt{7}\right)\left(\sqrt{7}+\sqrt{5}\right)}$$
$$= \frac{3\sqrt{7}\left(\sqrt{14}+\sqrt{10}+7+\sqrt{35}\right)-5\sqrt{5}\left(\sqrt{35}+5+\sqrt{14}+\sqrt{10}\right)+2\sqrt{2}\left(\sqrt{10}+\sqrt{35}+2+\sqrt{14}\right)}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{2}+\sqrt{7}\right)\left(\sqrt{7}+\sqrt{5}\right)}$$
$$= \frac{3\sqrt{7}\left(7+\sqrt{14}+\sqrt{10}+\sqrt{35}\right)-5\sqrt{5}\left(5+\sqrt{35}+\sqrt{14}+\sqrt{10}\right)+2\sqrt{2}\left(2+\sqrt{10}+\sqrt{35}+\sqrt{14}\right)}{2\sqrt{70}+12\sqrt{2}+9\sqrt{5}+7\sqrt{7}}$$
$$= \frac{0}{2\sqrt{70}+12\sqrt{2}+9\sqrt{5}+7\sqrt{7}}$$
$$= 0$$

[b] $x = 2 + \sqrt{3}$,

$y = 2 - \sqrt{3}$

$xy + 1 / xy$

$= (2 + \sqrt{3}) * (2 - \sqrt{3}) + 1 / [2 + \sqrt{3}] * [2 - \sqrt{3}]$

$= [2^2 - 3 + 1] / [2^2 - 3]$

$= 2 / 4 - 3$

$= 2 / 1$

$= 2$

Question 11: Answer any TWO questions: [5 x 2 = 10]

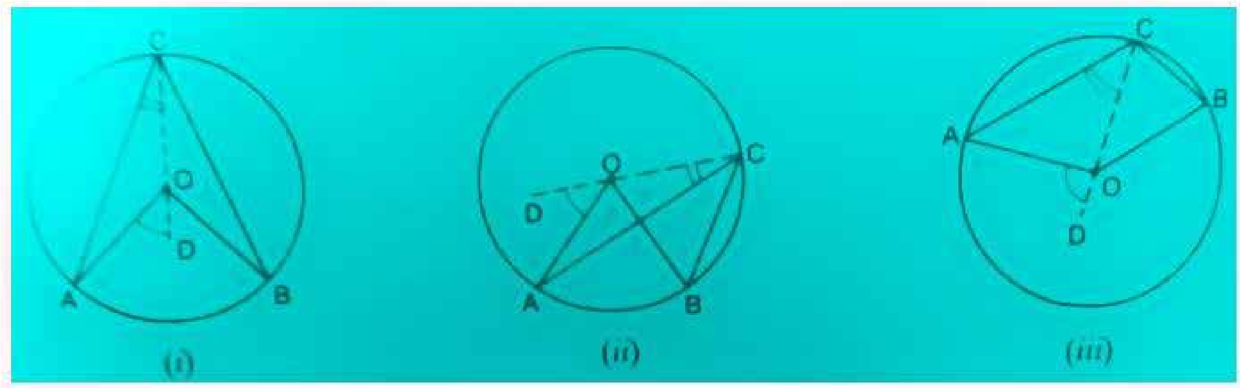
(a) Prove that the angle which an arc of a circle subtends at the centre is double the angle subtended by it at any point in the remaining part of the circle.

(b) Prove that a line segment drawn from the centre of a circle to bisect a chord which is not a diameter is at right angles to the chord.

(c) Prove that if a perpendicular is drawn from the vertex containing the right angle of a right-angled triangle on the hypotenuse the triangles on each side of the perpendicular are similar to each other.

Solution:

[a]



Let there be a circle with a centre O. Arc AB intends AOB at the centre and ACB at any point C on the remaining part of the circle.

Construction: Join CO and produce it to any point D.

Proof:

OA = OC [radii of same circle]

$\angle$OAC = $\angle$ACO[angles opp to equal side's of a triangle are equal]

$\angle$AOD = $\angle$OAC + $\angle$ACO [exterior angles = sum of equal opposite angles]

$\angle$AOD = 2($\angle$ACO)--------------(1) [$\angle$OAC = $\angle$ACO]

Similarly, $\angle$DOB = 2($\angle$OCB) --------------(2)

In fig (i) and (iii)

adding (1) And (2)

$\angle$AOD + $\angle$ DOB = 2($\angle$ACO) + 2($\angle$OCB)

$\angle$AOD + $\angle$ DOB = 2($\angle$ACO + $\angle$OCB)

$\angle$AOB = 2($\angle$ACB)

In fig (ii)

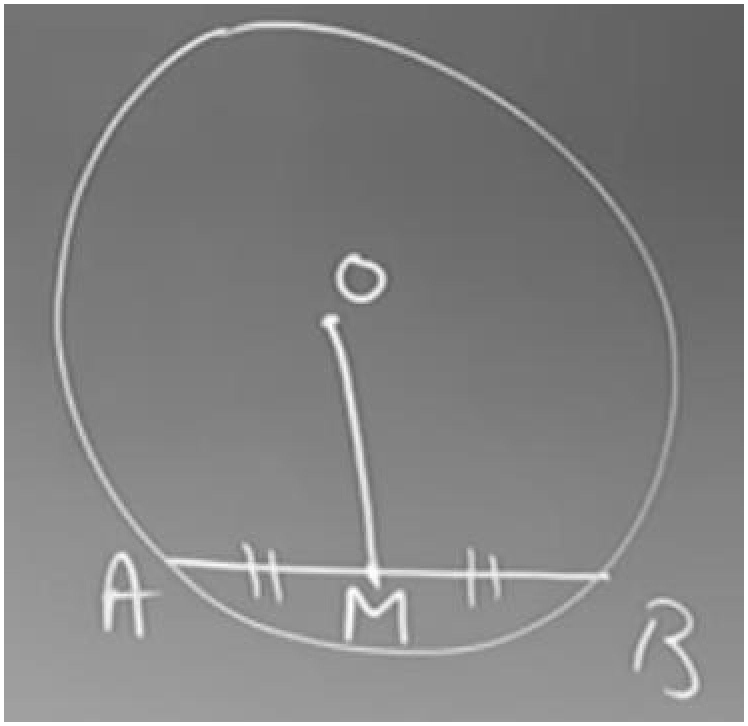
subtracting (1) from (2)

$\angle$DOB − $\angle$DOA = 2($\angle$OCB − $\angle$ACO)

$\angle$AOB = 2($\angle$ACB)

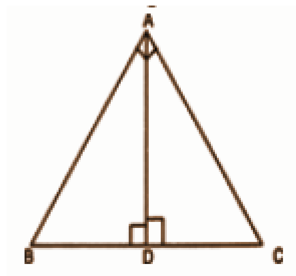
In all the cases, $\angle$AOB = 2($\angle$ACB)

[b]



Given O is the centre of the circle.
AB is the chord.
OM bisects AB at M.
AM = BM
Proof:
$OM \perp AB$
Join OA and OB
Consider $\triangle OAM$ and $\triangle OBM$
AM = BM [M is the midpoint of AB]
OM = OM [common]
OA = OB [radii of the same circle]
So, $\triangle OAM \cong \triangle OBM$
$\angle OMA = \angle OMP$ [CACT]
$\angle OMA + \angle OMB = 180^{o}$ [AB is a straight line]
$\angle OMA = \angle OMB = 90^{o}$

[c]



Given a right angle triangle, right-angled at A.
AD is the perpendicular drawn to the hypotenuse BC from vertex A.
To Prove:
(i) $\Delta BDA \sim \Delta BAC$
(ii) $\Delta ADC \sim \Delta BAC$
(iii) $\Delta BDA \sim AADC$
Proof: In ΔBDA and ΔBAC:
$\angle ADB = \angle A = 90°$
$\angle B = \angle B$ [common]
Therefore, by using AA similar condition
$\Delta BDA \sim \Delta BAC$ ...(i)
Now, in ΔADC and ΔBAC, we have
$\angle ADC = \angle A = 90°$
$\angle C = \angle C$ [common]
Therefore, by using AA similar condition
$\Delta ADC \sim \Delta BAC$ ...(ii)
Comparing (i) and (ii), we get
$\Delta BDA \sim \Delta ADC$.

Question 12: Answer any ONE question: [3 x 1 = 3]

(a) There are two concentric circles such that two chords AB and AC of greater circle touch the smaller circle at P and Q respectively. Prove that PQ = (1 / 2)BC.
(b) ABCD is a rectangle and O is any point within it. Prove that $OA^2 + OC^2 = OB^2 + OD^2$.

Solution:

[a] Given: O is the centre of two concentric circles.

To prove: PQ = (1 / 2) BC

Construction: Join OP & OQ.

PROOF: The perpendicular from the centre to the chord bisects the chord.

AB = 2AP & AC = 2AQ

AB / AC = AP / AQ

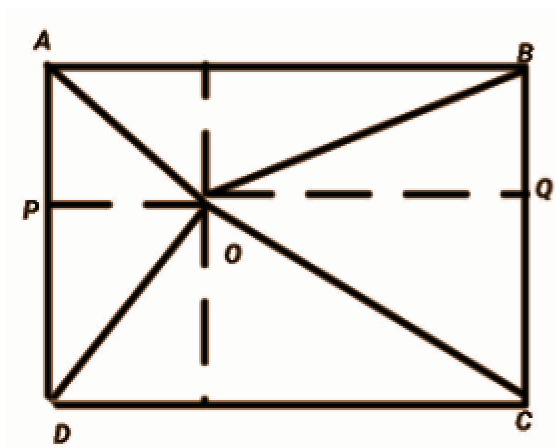
ΔAPQ is similar to ΔABC

AP / AB = PQ / BC

1 / 2 = PQ / BC

(1 / 2)BC = PQ

[b]



Draw PQ $\parallel$ AB $\parallel$ CD as shown in the figure.

ABCD is a rectangle, it means ABPQ and PQDC are also rectangles.

For ABPQ,

AP = BQ [opposite sides are equal]

For PQDC

PD = QC [opposite sides are equal]

For ΔOPD,

$OD^2 = OP^2 + PD^2$ ------(1)

For ΔOQB,

$OB^2 = OQ^2 + BQ^2$ --------(2)

Add equations (1) and (2),

$OB^2 + OD^2 = (OP^2 + PD^2) + (OQ^2 + BQ^2) = (OP^2 + CQ^2) + (OQ^2 + AP^2)$

In the figure, ΔOPA and ΔOQC are also right-angled triangles.

For, $\Delta OCQ \Rightarrow OQ^2 + CQ^2 = OC^2$

For, $\Delta OPA \Rightarrow OP^2 + AP^2 = OA^2$, put it above

Now,

$OB^2 + OD^2 = OC^2 + OA^2$

Question 13: Answer any ONE question: [5 x 1 = 5]

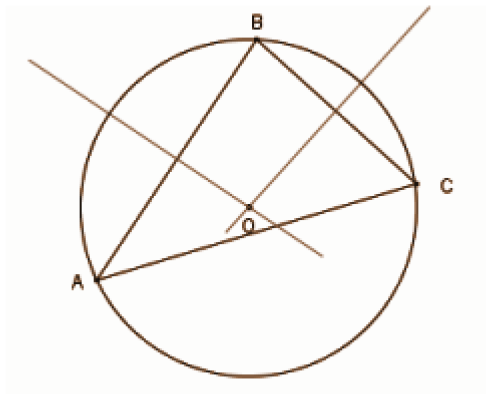
(a) Draw a circumcircle of a triangle. (Only traces of construction are required)

(b) Find geometrically the value of $\sqrt{21}$. (Only traces of construction are required)

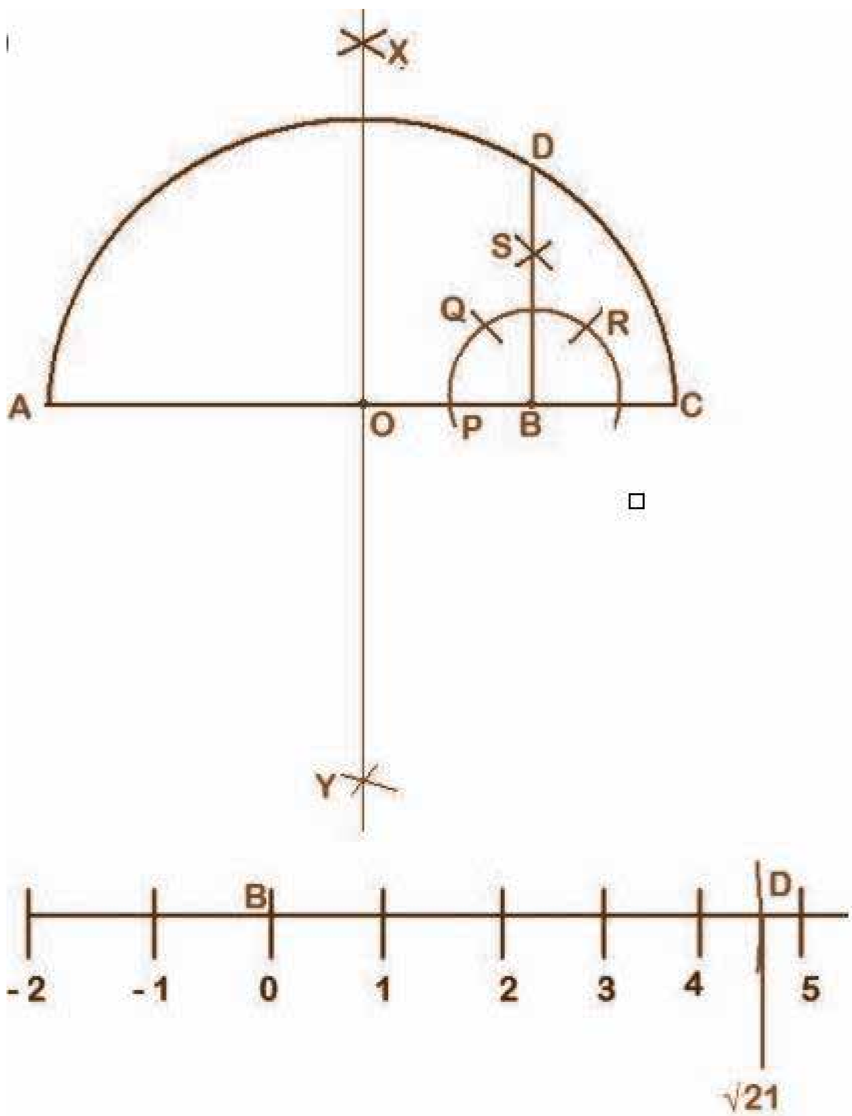
Solution:

[a] Start with a triangle ABC.

1. Find the bisector of one of the triangle sides.
2. Repeat for another side.
3. Repeat for the third side. Three bisectors exist which intersect at a single point.
4. The point where these two perpendiculars intersect is the triangle's circumcenter, the centre of the circle we desire.
4. Place the compasses' point on the intersection of the perpendiculars and set the compasses' width to one of the points A, B or C. Draw a circle that will pass through all three.
5. The circle drawn is the triangle's circumcircle, the only circle that will pass through all three of its vertices.



[b]



Step 1: Draw a line AB = 21 units.
Step 2: Now extend line AB, as BC = 1 unit.

Step 3: Take radius as more than half of AC and with centres, A and C. Draw two arcs from both points on both sides of line AC, these arcs intersect at X and Y.
Step 4: Join XY. Line XY intersects line AC at O. Take radius OA = OC and draw a semicircle.
Step 5: Take any radius (less than BC) and centre B, draw a semicircle that intersects line AC at P. Now with the same radius and centre P, draw an arc that intersects our semicircle at Q. With same radius and centre Q, draw an arc that intersects our semicircle at R. With same radius and centre Q and R, draw these arcs that intersect at S.
Step 6: Join BS and extend that line intersect to the main semicircle at D.
Step 7: Now on taking point B as the origin and with a radius of BD, draw an arc that intersects our number line at D.

Question 14: Answer any ONE question: [3 x 1 = 3]

(a) A hemisphere and a cone are on equal bases and their heights are also equal. Find the ratio of the area of their curved surfaces.
(b) The base of a right prism of height 10 cm is a rectangle whose length and breadth are 5 cm and 3 cm, respectively. Find the volume of the prism.

Solution:
[a] Base of a cone and hemisphere are equal then their radius is also equal.
Radius and heights of the hemisphere and cone are the same.
Let the radius of the hemisphere = radius of the cone = r
Height of the cone = radius of the hemisphere = r
Slant height of a cone, $l = \sqrt{r^2 + h^2}$
$l = \sqrt{r^2 + r^2}$
$l = \sqrt{2r^2}$
$l = r\sqrt{2}$ ………….(1)
The curved surface area of the hemisphere (S1) / Curved surface area of the cone (S2) = $2\pi r^2 / \pi rl$
S1 / S2 = $2\pi r^2 / \pi rl$
S1 / S2 = $2r / l$
S1 / S2 = $2r / r\sqrt{2}$ [From equation 1]

S1 / S2 = 2 / √2

S1 / S2 = (2 × √2) / (√2 × √2) [On rationalising the denominator]

S1 / S2 = 2√2 / 2

S1 / S2 = √2 / 1

S1 : S2 = √2 : 1

The ratio of the curved surface area of the hemisphere and cone = √2:1.

[b] Area of base = L * B

= 5 * 3

= 15cm^2

The volume of the prism = base area * height

= 15 * 10

= 150 cm^3

Question 15: Answer any ONE question: [4 x 1 = 4]

(a) 77 square metre tarpaulin needed to make a right circular conical tent. If the slant height of the tent is 7 m, find the area of the base of the tent.

(b) By melting two solid spheres of radii 1 cm and 6 cm, a hollow sphere of thickness 1 cm is made. Find the outer curved surface area of the new sphere.

Solution:

[a] Structure of a tent is like a right circular cone.

Tarpaulin required to make tent = curved surface area of the cone

Let, the base radius of the tent = 'r' m

Slant height = 7 m

⇒ π × r × 7 = 77

⇒ 22 / 7 × r = 11

⇒ r = 7 / 2 = 3.5

∴ Base radius of the tent = 3.5 m

∴ The base area of the tent = πr^2

$= 22 / 7 \times 3.5^2$
$= 38.5$ sq. m

[b] The volume of sphere 1 = $(4 / 3)(22 / 7) * 1^3 = (88 / 21) * 1$ cc
The volume of sphere 2 = $(4 / 3)(22 / 7) * 6^3 = (88 / 21) * 216$ cc.
The combined volume of the two spheres = $(88 / 21) * 217$ cc.
The hollow sphere has an outer radius of x and the internal radius = (x - 1).
Volume of the hollow sphere
$= (4 / 3)(22 / 7)[x^3 - (x - 1)^3]$
$= (88 / 21)[x^3 - (x - 1)^3]$
$= (88 / 21) * 217$
$[x^3 - (x - 1)^3] = 217$
$x^3 - (x^3 - 3x^2 + 3x - 1) = 217$
$x^3 - x^3 + 3x^2 - 3x + 1 = 217$
$3x^2 - 3x - 216 = 0$
$x^2 - x - 72 = 0$
$(x - 9)(x + 8) = 0$
x = 9 as -8 as negative value cannot be taken.
So the hollow sphere has a surface area of $4 * \pi * 9^2$
= 1018.29 sq cm.

Question 16: Answer any TWO questions: [3 x 2 = 6]

(a) A rotating ray revolves in the anticlockwise direction and makes two complete revolutions from its initial position and moves further to trace an angle of 30°. What are the sexagesimal and circular measures of the angle with reference to trigonometrical measure?
(b) If β and α are complementary angles to each other, then find the value of $(1 - \sin^2\alpha)(1 - \cos^2\alpha)(1 + \cot^2\beta)(1 + \tan^2\beta)$.
(c) Prove that $\sqrt{1 + \cos 30^\circ / 1 - \cos 30^\circ} = \sec 60^\circ + \tan 60^\circ$.
(d) If $\cos 52^\circ = x / \sqrt{x^2 + y^2}$, then find the value of tan 38°.

Solution:
[a] One complete revolution = 360°.

So in two complete revolutions, it makes an angle of $360° \times 2 = 720°$.
$30°$ is traced by moving further.
The magnitude of the angle formed is $(720° + 30°)$
$= 750°$.
$= 750 \pi / 180$
$= 25 \pi / 6$

[b] $(1 - \sin^2\alpha) (1 - \cos^2\alpha) (1 + \cot^2\beta) (1 + \tan^2\beta)$
$= \cos^2 \alpha * \sin^2 \alpha * \mathrm{cosec}^2 \beta * \sec^2 \beta$
$\alpha + \beta = 90^o$
$\beta = 90 - \alpha$
$\cos \beta = \cos (90 - \alpha)$
$\cos \beta = \sin \alpha$
$\beta = 90 - \alpha$
$\sin \beta = \sin (90 - \alpha)$
$\sin \beta = \cos\alpha$
$= \cos \alpha * \cos \alpha * \sin \alpha * \sin \alpha * (1 / \sin^2 \beta) * (1 / \cos^2 \beta)$
$= \cos \alpha * \cos \alpha * \sin \alpha * \sin \alpha * (1 / \sin \alpha * \sin \alpha) * (1 / \cos \alpha * \cos \alpha)$
$= 1$

[c] $\sqrt{1 + \cos 30^o / 1 - \cos 30^o}$
$= (\sqrt{1 + \cos 30^o / 1 - \cos 30^o}) * (\sqrt{1 + \cos 30^o / 1 + \cos 30^o})$
$= (\sqrt{(1 + \cos^2 30^o)^2 / (1 - \cos^2 30^o)}$
$= (\sqrt{1 + \cos 30^o / \sin 30^o}$
$= \mathrm{cosec}\ 30^o + \cot 30^o$

[d] $\cos 52^o = x / \sqrt{x^2 + y^2}$
$\tan 38^o = ?$
$\cos (90^o - 52^o) = x / \sqrt{x^2 + y^2}$
$\sin \alpha = P / H$
By the Pythagoras theorem,
$H^2 = P^2 + B^2$
$B = \sqrt{H^2 - P^2}$

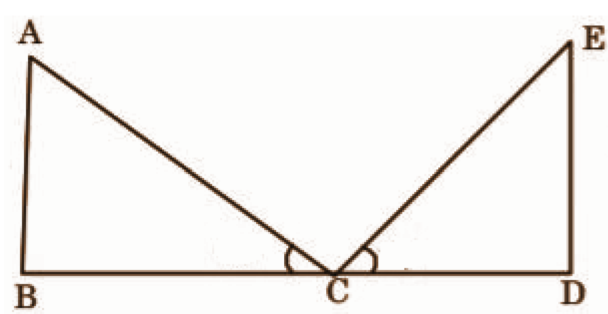
$= \sqrt{x^2 + y^2 - x^2}$
$= y$
$\tan 38^0 = P / B = x / y$

Question 17: Answer any ONE question: [5 x 1 = 5]

(a) From a quay of a river, 600 metres wide, two boats start in two different directions to reach the opposite side of the river. The first boat moves to make an angle of 30° with the bank and the second boat moves to make an angle of 90° with the direction of the first board. What will be the distance between the two boats when both of them reach the other side?
(b) The height of the two towers is h_1 and h_2 respectively. If the angle of elevation of the top of the first tower from the foot of the second tower is 60°, and the angle of elevation of the top of the second tower from the foot of the first tower is 45°, then show that $h_1^2 = 3h_2^2$.

Solution:
[a] tan 30° = width of river/distance between the two boats when both of them reach the other side
=> $1 / \sqrt{3}$ = 600 / distance between the two boats when both of them reach the other side
=> distance between the two boats when both of them reach the other side = $600\sqrt{3}$ m
= 1039.2 m
1039.2 m will be the distance between the two boats when both of them reach the other side.

[b]



$\tan 60 = AB / BC$

$\sqrt{3} = H_1 / x$

$x = H_1 / \sqrt{3}$

$\tan 45 = DE / DC$

$1 = H_2 / x$

$x = H_2$

$H_1 / \sqrt{3} = H_2$

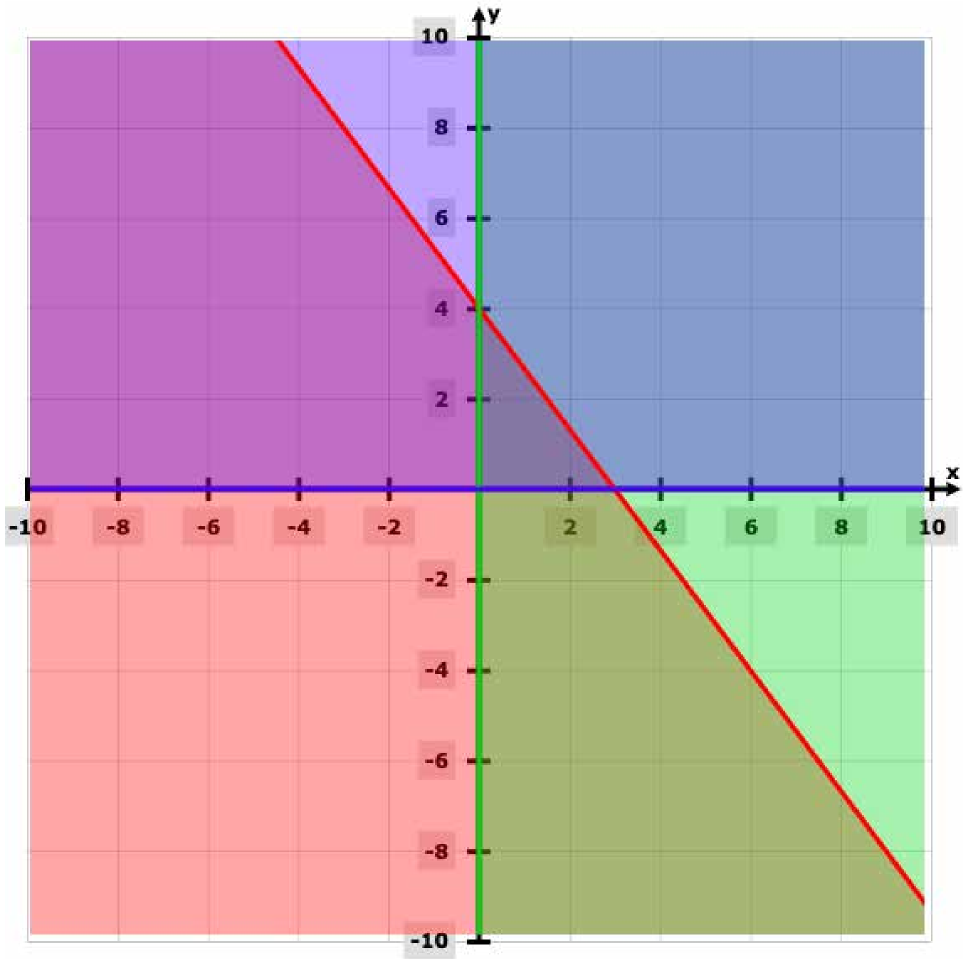
Squaring both sides,

$H_1^2 = 3H_2^2$

Question 18: **[Alternative Questions for Sightless Candidates]**

Find the coordinates of three points lying in the solution region satisfying the inequalities: $4x + 3y \le 12$; $x \ge 0$, $y \ge 0$. **[4M]**

Solution:

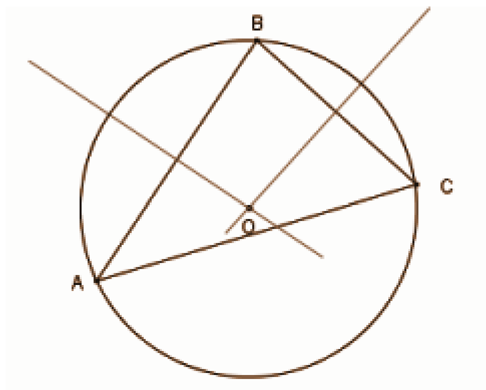


(2, 1), (0, 4) and (3, 0) are the coordinates of three points lying in the solution region satisfying the given inequalities.

Question 19: Answer any ONE question: [5 x 1 = 5]
(a) Describing the procedure of constructing the circumcircle of a triangle whose length of three sides are given?
(b) Describe the procedure of drawing a tangent at a point on a circle.

Solution:
[a]
(i) Start with a triangle ABC.
1. Find the bisector of one of the triangle sides.
2. Repeat for another side.
3. Repeat for the third side. three bisectors exist which intersect at a single point.

4. The point where these two perpendiculars intersect is the triangle's circumcenter, the centre of the circle we desire.
4. Place the compasses' point on the intersection of the perpendiculars and set the compasses' width to one of the points A, B or C. Draw a circle that will pass through all three.
5. The circle drawn is the triangle's circumcircle, the only circle that will pass through all three of its vertices.



[b] Construction of a tangent to a circle
To draw a tangent to a circle at a point on the circle. (Refer fig.)
Step 1: Draw a circle with the required radius with centre O.
Step 2: Join the centre of the circle O and any point P on the circle. OP is the radius of the circle.
Step 3: Draw a line perpendicular to radius OP through point P. This line will be a tangent to the circle at P.

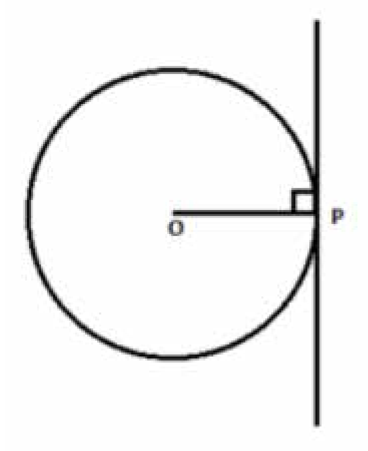


Figure 1-construction of tangent

[Additional Question for External Candidates]

Question 20: Answer ALL Questions:

(a) 75% of A = 50% of B; find A:B. **[2]**

(b) Find the mean proportion of $6a^3b$ and $24ab^3$ **[1]**

(c) If tan $(\theta + 15^o) = \sqrt{3}$, find sin θ.

[2]

(d) Find the value of π / 12 radians in degree. **[1]**

(e) If the length of each side of a rhombus is 5 cm and the length of one diagonal is 8 cm, find the length of the other diagonal. **[2]**

(f) How many solid right circular cones can be formed by melting a solid cylinder having equal radius and height? **[2]**

Solution:

[a] 75% of A = 50% of B.

[75 / 100] * A = [50 / 100] * B

75A / 100 = 50B / 100

3A / 4 = B / 2

6A = 4B

6 / 4 = A / B

3 / 2 = A / B

A:B = 3:2

[b] $6a^3b$ and $24ab^3$

Mean proportion = $(6a^3b * 24ab^3)^{½}$

$= (144\, a^4b^4)^{½}$

$= 12a^2b^2$

[c] $\tan(\theta + 15^o) = \sqrt{3}$

$(\theta + 15^o) = \tan^{-1}(\sqrt{3})$

$\theta + 15^o = 60$

$\theta = 60^o - 15^o$

$\theta = 45^o$

[d] π / 12 radians

$= x * (180 / \pi)$

$= [\pi / 12] * (180 / \pi)$

$= 180^o / 12$

$= 15^o$

[e] The diagonals of a rhombus intersect at the right angle.

⇒ Half of each diagonal and the length of the side will form a right angle triangle.

Diagonal = 8 cm

1/2 the diagonal = 8 ÷ 2 = 4 cm

$a^2 + b^2 = c^2$

$a^2 + 4^2 = 5^2$

$a^2 = 5^2 - 4^2$

$a^2 = 9$

$a = \sqrt{9}$

a = 3 cm

1 / 2 the diagonal = 3 cm

Diagonal = 3 x 2 = 6 cm

The length of the other diagonal is 6 cm.

[f] Two solid right circular cones.