## Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

## Special Instructions / Useful Data

| | |
|---|---|
| $\mathbb{R}$ | The set of all real numbers |
| $P^T$ | Transpose of the matrix $P$ |
| $\mathbb{R}^n$ | $\left\{ \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} : x_i \in \mathbb{R}, i = 1, 2, \dots, n \right\}$ |
| $f'$ | Derivative of the differentiable function $f$ |
| $I_n$ | $n \times n$ identity matrix |
| $P(E)$ | Probability of the event $E$ |
| $E(X)$ | Expectation of the random variable $X$ |
| $Var(X)$ | Variance of the random variable $X$ |
| i.i.d. | Independently and identically distributed |
| $U(a, b)$ | Continuous uniform distribution on $(a, b)$, $-\infty < a < b < \infty$ |
| $Exp(\lambda)$ | Exponential distribution with probability density function, for $\lambda > 0$, $$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0, \\ 0, & \text{otherwise} \end{cases}$$ |
| $N(\mu, \sigma^2)$ | Normal distribution with mean $\mu$ and variance $\sigma^2$ |
| $\Phi(a)$ | $\dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\infty}^{a} e^{-\frac{u^2}{2}} du$ |
| $\chi_n^2$ | Central Chi-squared distribution with $n$ degrees of freedom |
| $t_{n, \alpha}$ | A constant such that $P\left(X > t_{n, \alpha}\right) = \alpha$, where $X$ has Student's $t$-distribution with $n$ degrees of freedom |
| $n!$ | $n(n-1)\cdots 3 \cdot 2 \cdot 1$ for $n = 1, 2, 3 \dots$, and $0! = 1$ |
| | $\Phi(1.65) = 0.950, \quad \Phi(1.96) = 0.975$ <br> $t_{4, 0.05} = 2.132, \qquad t_{4, 0.10} = 1.533$ |

## SECTION – A
## MULTIPLE CHOICE QUESTIONS (MCQ)

### Q. 1 – Q.10 carry one mark each.

Q.1  Let $\{x_n\}_{n\geq 1}$ be a sequence of positive real numbers. Which one of the following statements is always TRUE?

(A) If $\{x_n\}_{n\geq 1}$ is a convergent sequence, then $\{x_n\}_{n\geq 1}$ is monotone

(B) If $\{x_n^2\}_{n\geq 1}$ is a convergent sequence, then the sequence $\{x_n\}_{n\geq 1}$ does not converge

(C) If the sequence $\{|x_{n+1} - x_n|\}_{n\geq 1}$ converges to 0, then the series $\sum_{n=1}^{\infty} x_n$ is convergent

(D) If $\{x_n\}_{n\geq 1}$ is a convergent sequence, then $\{e^{x_n}\}_{n\geq 1}$ is also a convergent sequence

Q.2  Consider the function $f(x, y) = x^3 - 3xy^2$, $x, y \in \mathbb{R}$. Which one of the following statements is TRUE?

(A) $f$ has a local minimum at $(0, 0)$

(B) $f$ has a local maximum at $(0, 0)$

(C) $f$ has global maximum at $(0, 0)$

(D) $f$ has a saddle point at $(0, 0)$

Q.3  If $F(x) = \int_{x^3}^{4} \sqrt{4 + t^2}\, dt$, for $x \in \mathbb{R}$, then $F'(1)$ equals

(A) $-3\sqrt{5}$      (B) $-2\sqrt{5}$      (C) $2\sqrt{5}$      (D) $3\sqrt{5}$

Q.4  Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $T\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.
Suppose that $\begin{bmatrix} 3 \\ -2 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \beta \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $T\left(\begin{bmatrix} 3 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} a \\ b \end{bmatrix}$. Then $\alpha + \beta + a + b$ equals

(A) $\dfrac{2}{3}$      (B) $\dfrac{4}{3}$      (C) $\dfrac{5}{3}$      (D) $\dfrac{7}{3}$

Q.5  Two biased coins $C_1$ and $C_2$ have probabilities of getting heads $\dfrac{2}{3}$ and $\dfrac{3}{4}$, respectively, when tossed. If both coins are tossed independently two times each, then the probability of getting exactly two heads out of these four tosses is

(A) $\dfrac{1}{4}$      (B) $\dfrac{37}{144}$      (C) $\dfrac{41}{144}$      (D) $\dfrac{49}{144}$

Q.6 Let $X$ be a discrete random variable with the probability mass function

$$P(X = n) = \begin{cases} \dfrac{-2c}{n}, & n = -1, -2, \\ d, & n = 0, \\ cn, & n = 1, 2, \\ 0, & \text{otherwise,} \end{cases}$$

where $c$ and $d$ are positive real numbers. If $P(|X| \leq 1) = 3/4$, then $E(X)$ equals

(A) $\dfrac{1}{12}$      (B) $\dfrac{1}{6}$      (C) $\dfrac{1}{3}$      (D) $\dfrac{1}{2}$

Q.7 Let $X$ be a Poisson random variable and $P(X = 1) + 2\,P(X = 0) = 12\,P(X = 2)$. Which one of the following statements is TRUE?

(A) $0.40 < P(X = 0) \leq 0.45$      (B) $0.45 < P(X = 0) \leq 0.50$

(C) $0.50 < P(X = 0) \leq 0.55$      (D) $0.55 < P(X = 0) \leq 0.60$

Q.8 Let $X_1, X_2, \ldots$ be a sequence of i.i.d. discrete random variables with the probability mass function

$$P(X_1 = m) = \begin{cases} \dfrac{(\log_e 2)^m}{2(m!)}, & m = 0, 1, 2, \ldots, \\ 0, & \text{otherwise.} \end{cases}$$

If $S_n = X_1 + X_2 + \cdots + X_n$, then which one of the following sequences of random variables converges to 0 in probability?

(A) $\dfrac{S_n}{n \log_e 2}$      (B) $\dfrac{S_n - n \log_e 2}{n}$      (C) $\dfrac{S_n - \log_e 2}{n}$      (D) $\dfrac{S_n - n}{\log_e 2}$

Q.9 Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution with the probability density function

$$f(x) = \frac{1}{2\sqrt{2\pi}} \left[ e^{-\frac{1}{2}(x - 2\mu)^2} + e^{-\frac{1}{2}(x - 4\mu)^2} \right], \quad -\infty < x < \infty.$$

If $T = X_1 + X_2 + \cdots + X_n$, then which one of the following is an unbiased estimator of $\mu$?

(A) $\dfrac{T}{n}$      (B) $\dfrac{T}{2n}$      (C) $\dfrac{T}{3n}$      (D) $\dfrac{T}{4n}$

Q.10  Let $X_1, X_2, \ldots, X_n$ be a random sample from a $N(\theta, 1)$ distribution. Instead of observing $X_1, X_2, \ldots, X_n$, we observe $Y_1, Y_2, \ldots, Y_n$, where $Y_i = e^{X_i}$, $i = 1, 2, \ldots, n$. To test the hypothesis

$$H_0 : \theta = 1 \text{ against } H_1 : \theta \neq 1$$

based on the random sample $Y_1, Y_2, \ldots, Y_n$, the rejection region of the likelihood ratio test is of the form, for some $c_1 < c_2$,

(A) $\sum_{i=1}^n Y_i \leq c_1$  or  $\sum_{i=1}^n Y_i \geq c_2$ 　　　　　 (B) $c_1 \leq \sum_{i=1}^n Y_i \leq c_2$

(C) $c_1 \leq \sum_{i=1}^n \log_e Y_i \leq c_2$ 　　　　　 (D) $\sum_{i=1}^n \log_e Y_i \leq c_1$  or  $\sum_{i=1}^n \log_e Y_i \geq c_2$

## Q. 11 – Q. 30 carry two marks each.

Q.11  $\sum_{n=4}^{\infty} \dfrac{6}{n^2 - 4n + 3}$ equals

(A) $\dfrac{5}{2}$ 　　　　　 (B) 3 　　　　　 (C) $\dfrac{7}{2}$ 　　　　　 (D) $\dfrac{9}{2}$

Q.12  $\displaystyle \lim_{n \to \infty} \dfrac{1 + \frac{1}{2} + \cdots + \frac{1}{n}}{(\pi^n + e^n)^{1/n} \; \log_e n}$ equals

(A) $\dfrac{1}{\pi}$ 　　　　　 (B) $\dfrac{1}{e}$ 　　　　　 (C) $\dfrac{e}{\pi}$ 　　　　　 (D) $\dfrac{\pi}{e}$

Q.13  A possible value of $b \in \mathbb{R}$ for which the equation $x^4 + bx^3 + 1 = 0$ has no real root is

(A) $\dfrac{-11}{5}$ 　　　　　 (B) $\dfrac{-3}{2}$ 　　　　　 (C) 2 　　　　　 (D) $\dfrac{5}{2}$

Q.14  Let the Taylor polynomial of degree $20$ for $\dfrac{1}{(1-x)^3}$ at $x = 0$ be $\sum_{n=0}^{20} a_n x^n$. Then $a_{15}$ is

(A) 136 　　　　　 (B) 120 　　　　　 (C) 60 　　　　　 (D) 272

Q.15  The length of the curve $y = \dfrac{3}{4} x^{4/3} - \dfrac{3}{8} x^{2/3} + 7$ from $x = 1$ to $x = 8$ equals

(A) $\dfrac{99}{8}$ 　　　　　 (B) $\dfrac{117}{8}$ 　　　　　 (C) $\dfrac{99}{4}$ 　　　　　 (D) $\dfrac{117}{4}$

Q.16    The volume of the solid generated by revolving the region bounded by the parabola $x = 2y^2 + 4$ and the line $x = 6$ about the line $x = 6$ is

(A) $\dfrac{78\pi}{15}$         (B) $\dfrac{91\pi}{15}$         (C) $\dfrac{64\pi}{15}$         (D) $\dfrac{117\pi}{15}$

Q.17    Let $P$ be a $3 \times 3$ non-null real matrix. If there exist a $3 \times 2$ real matrix $Q$ and a $2 \times 3$ real matrix $R$ such that $P = QR$, then

(A) $P\boldsymbol{x} = \mathbf{0}$ has a unique solution, where $\mathbf{0} \in \mathbb{R}^3$
(B) there exists $\boldsymbol{b} \in \mathbb{R}^3$ such that $P\boldsymbol{x} = \boldsymbol{b}$ has no solution
(C) there exists a non-zero $\boldsymbol{b} \in \mathbb{R}^3$ such that $P\boldsymbol{x} = \boldsymbol{b}$ has a unique solution
(D) there exists a non-zero $\boldsymbol{b} \in \mathbb{R}^3$ such that $P^T\boldsymbol{x} = \boldsymbol{b}$ has a unique solution

Q.18    If $P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 2 & 0 & -1 \end{bmatrix}$ and $6P^{-1} = aI_3 + bP - P^2$, then the ordered pair $(a, b)$ is

(A) $(3, 2)$            (B) $(2, 3)$            (C) $(4, 5)$            (D) $(5, 4)$

Q.19    Let $E, F$ and $G$ be any three events with $P(E) = 0.3$, $P(F|E) = 0.2$, $P(G|E) = 0.1$ and $P(F \cap G|E) = 0.05$. Then $P(E - (F \cup G))$ equals

(A) $0.155$            (B) $0.175$            (C) $0.225$            (D) $0.255$

Q.20    Let $E$ and $F$ be any two independent events with $0 < P(E) < 1$ and $0 < P(F) < 1$. Which one of the following statements is **NOT** TRUE?

(A) $P(\text{Neither } E \text{ nor } F \text{ occurs}) = (P(E) - 1)(P(F) - 1)$
(B) $P(\text{Exactly one of } E \text{ and } F \text{ occurs}) = P(E) + P(F) - P(E)P(F)$
(C) $P(E \text{ occurs but } F \text{ does not occur}) = P(E) - P(E \cap F)$
(D) $P(E \text{ occurs given that } F \text{ does not occur}) = P(E)$

Q.21    Let $X$ be a continuous random variable with the probability density function

$$f(x) = \begin{cases} \dfrac{1}{3}\, x^7 e^{-x^2}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Then the distribution of the random variable $W = 2X^2$ is

(A) $\chi^2_2$            (B) $\chi^2_4$            (C) $\chi^2_6$            (D) $\chi^2_8$

Q.22   Let $X$ be a continuous random variable with the probability density function
$$f(x) = \frac{e^x}{(1 + e^x)^2}, \quad -\infty < x < \infty.$$
Then $E(X)$ and $P(X > 1)$, respectively, are

(A) $1$ and $(1 + e)^{-1}$                          (B) $0$ and $2(1 + e)^{-2}$
(C) $2$ and $(2 + 2e)^{-1}$                          (D) $0$ and $(1 + e)^{-1}$


Q.23   The lifetime (in years) of bulbs is distributed as an $Exp(1)$ random variable. Using Poisson approximation to the binomial distribution, the probability (round off to 2 decimal places) that out of the fifty randomly chosen bulbs at most one fails within one month equals

(A) $0.05$            (B) $0.07$            (C) $0.09$            (D) $0.11$


Q.24   Let $X$ follow a beta distribution with parameters $m$ $(> 0)$ and $2$. If $P\left(X \le \frac{1}{2}\right) = \frac{1}{2}$, then $Var(X)$ equals

(A) $\dfrac{1}{10}$            (B) $\dfrac{1}{20}$            (C) $\dfrac{1}{25}$            (D) $\dfrac{1}{40}$


Q.25   Let $X_1, X_2$ and $X_3$ be i.i.d. $U(0,1)$ random variables. Then $P(X_1 > X_2 + X_3)$ equals

(A) $\dfrac{1}{6}$            (B) $\dfrac{1}{4}$            (C) $\dfrac{1}{3}$            (D) $\dfrac{1}{2}$


Q.26   Let $X$ and $Y$ be i.i.d. $U(0, 1)$ random variables. Then $E(X|X > Y)$ equals

(A) $\dfrac{1}{3}$            (B) $\dfrac{1}{2}$            (C) $\dfrac{2}{3}$            (D) $\dfrac{3}{4}$


Q.27   Let $-1$ and $1$ be the observed values of a random sample of size two from $N(\theta, \theta)$ distribution. The maximum likelihood estimate of $\theta$ is

(A) $0$            (B) $2$            (C) $\dfrac{-\sqrt{5}-1}{2}$            (D) $\dfrac{\sqrt{5}-1}{2}$

Q.28    Let $X_1$ and $X_2$ be a random sample from a continuous distribution with the probability density function

$$f(x) = \begin{cases} \dfrac{1}{\theta} \, e^{-\frac{x-\theta}{\theta}}, & x > \theta, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta > 0$. If $X_{(1)} = \min\{X_1, X_2\}$ and $\overline{X} = \dfrac{(X_1 + X_2)}{2}$, then which one of the following statements is TRUE?

(A) $\left(\overline{X}, X_{(1)}\right)$ is sufficient and complete

(B) $\left(\overline{X}, X_{(1)}\right)$ is sufficient but not complete

(C) $\left(\overline{X}, X_{(1)}\right)$ is complete but not sufficient

(D) $\left(\overline{X}, X_{(1)}\right)$ is neither sufficient nor complete


Q.29    Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution with the probability density function $f(x)$. To test the hypothesis

$$H_0 : f(x) = \frac{1}{\sqrt{\pi}} \, e^{-x^2}, -\infty < x < \infty \text{ against } H_1 : f(x) = e^{-2|x|}, -\infty < x < \infty,$$

the rejection region of the most powerful size $\alpha$ test is of the form, for some $c > 0$,

(A) $\sum_{i=1}^{n}(X_i - 1)^2 \geq c$                     (B) $\sum_{i=1}^{n}(X_i - 1)^2 \leq c$

(C) $\sum_{i=1}^{n}(|X_i| - 1)^2 \geq c$                   (D) $\sum_{i=1}^{n}(|X_i| - 1)^2 \leq c$


Q.30    Let $X_1, X_2, \ldots, X_n$ be a random sample from a $N(\theta, 1)$ distribution. To test $H_0 : \theta = 0$ against $H_1 : \theta = 1$, assume that the critical region is given by $\dfrac{1}{n}\sum_{i=1}^{n} X_i > \dfrac{3}{4}$. Then the minimum sample size required so that $P(\text{Type I error}) \leq 0.05$ is

(A) 3                  (B) 4                  (C) 5                  (D) 6

## SECTION - B

## MULTIPLE SELECT QUESTIONS (MSQ)

## Q. 31 – Q. 40 carry two marks each.

Q.31   Let $\{x_n\}_{n\geq 1}$ be a sequence of positive real numbers such that the series $\sum_{n=1}^{\infty} x_n$ converges. Which of the following statements is (are) always TRUE?

(A) The series $\sum_{n=1}^{\infty} \sqrt{x_n x_{n+1}}$ converges

(B) $\lim_{n\to\infty} n\, x_n = 0$

(C) The series $\sum_{n=1}^{\infty} \sin^2 x_n$ converges

(D) The series $\sum_{n=1}^{\infty} \dfrac{\sqrt{x_n}}{1+\sqrt{x_n}}$ converges

Q.32   Let $f : \mathbb{R} \to \mathbb{R}$ be continuous on $\mathbb{R}$ and differentiable on $(-\infty, 0) \cup (0, \infty)$. Which of the following statements is (are) always TRUE?

(A) If $f$ is differentiable at $0$ and $f'(0) = 0$, then $f$ has a local maximum or a local minimum at $0$

(B) If $f$ has a local minimum at $0$, then $f$ is differentiable at $0$ and $f'(0) = 0$

(C) If $f'(x) < 0$ for all $x < 0$ and $f'(x) > 0$ for all $x > 0$, then $f$ has a global maximum at $0$

(D) If $f'(x) > 0$ for all $x < 0$ and $f'(x) < 0$ for all $x > 0$, then $f$ has a global maximum at $0$

Q.33   Let $P$ be a $2 \times 2$ real matrix such that every non-zero vector in $\mathbb{R}^2$ is an eigenvector of $P$. Suppose that $\lambda_1$ and $\lambda_2$ denote the eigenvalues of $P$ and $P\begin{bmatrix} \sqrt{2} \\ \sqrt{3} \end{bmatrix} = \begin{bmatrix} 2 \\ t \end{bmatrix}$ for some $t \in \mathbb{R}$. Which of the following statements is (are) TRUE?

(A) $\lambda_1 \neq \lambda_2$

(B) $\lambda_1 \lambda_2 = 2$

(C) $\sqrt{2}$ is an eigenvalue of $P$

(D) $\sqrt{3}$ is an eigenvalue of $P$

Q.34   Let $P$ be an $n \times n$ non-null real skew-symmetric matrix, where $n$ is even. Which of the following statements is (are) always TRUE?

(A) $P\boldsymbol{x} = \boldsymbol{0}$ has infinitely many solutions, where $\boldsymbol{0} \in \mathbb{R}^n$

(B) $P\boldsymbol{x} = \lambda\boldsymbol{x}$ has a unique solution for every non-zero $\lambda \in \mathbb{R}$

(C) If $Q = (I_n + P)(I_n - P)^{-1}$, then $Q^T Q = I_n$

(D) The sum of all the eigenvalues of $P$ is zero

Q.35   Let $X$ be a random variable with the cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0, \\ \dfrac{1 + x^2}{10}, & 0 \le x < 1, \\ \dfrac{3 + x^2}{10}, & 1 \le x < 2, \\ 1, & x \ge 2. \end{cases}$$

Which of the following statements is (are) TRUE?

(A)  $P(1 < X < 2) = \dfrac{3}{10}$

(B)  $P(1 < X \le 2) = \dfrac{3}{5}$

(C)  $P(1 \le X < 2) = \dfrac{1}{2}$

(D)  $P(1 \le X \le 2) = \dfrac{4}{5}$


Q.36   Let $X$ and $Y$ be i.i.d. $Exp(\lambda)$ random variables. If $Z = \max\{X - Y, 0\}$, then which of the following statements is (are) TRUE?

(A)  $P(Z = 0) = \dfrac{1}{2}$

(B)  The cumulative distribution function of $Z$ is $F(z) = \begin{cases} 0, & z < 0, \\ 1 - \dfrac{1}{2}e^{-\lambda z}, & z \ge 0 \end{cases}$

(C)  $P(Z = 0) = 0$

(D)  The cumulative distribution function of $Z$ is $F(z) = \begin{cases} 0, & z < 0, \\ 1 - e^{-\lambda z/2}, & z \ge 0 \end{cases}$


Q.37   Let the discrete random variables $X$ and $Y$ have the joint probability mass function

$$P(X = m, Y = n) = \begin{cases} \dfrac{e^{-2}}{m!\,n!}, & m = 0,1,2, \dots;\ n = 0,1,2, \dots, \\ 0, & \text{otherwise.} \end{cases}$$

Which of the following statements is (are) TRUE?

(A)  The marginal distribution of $X$ is Poisson with mean 2
(B)  The random variables $X$ and $Y$ are independent
(C)  The covariance between $X$ and $X + \sqrt{3}\,Y$ is 1
(D)  $P(Y = n) = (n + 1)P(Y = n + 1)$ for $n = 0,1,2, \dots$

Q.38 Let $X_1, X_2, \ldots$ be a sequence of i.i.d. continuous random variables with the probability density function

$$f(x) = \begin{cases} 2e^{-2\left(x - \frac{1}{2}\right)}, & x \geq \frac{1}{2}, \\ 0, & \text{otherwise.} \end{cases}$$

If $S_n = X_1 + X_2 + \cdots + X_n$ and $\overline{X}_n = S_n/n$, then the distributions of which of the following sequences of random variables converge(s) to a normal distribution with mean 0 and a finite variance?

(A) $\dfrac{S_n - n}{\sqrt{n}}$          (B) $\dfrac{S_n}{\sqrt{n}}$          (C) $\sqrt{n}\left(\overline{X}_n - 1\right)$     (D) $\dfrac{\sqrt{n}\left(\overline{X}_n - 1\right)}{2}$

Q.39 Let $X_1, X_2, \ldots, X_n$ be a random sample from a $U(\theta, 0)$ distribution, where $\theta < 0$. If $T_n = \min\{X_1, X_2, \ldots, X_n\}$, then which of the following sequences of estimators is (are) consistent for $\theta$?

(A) $T_n$          (B) $T_n - 1$          (C) $T_n + \dfrac{1}{n}$          (D) $T_n - 1 - \dfrac{1}{n^2}$

Q.40 Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution with the probability density function, for $\lambda > 0$,

$$f(x) = \begin{cases} 2\lambda x e^{-\lambda x^2}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

To test the hypothesis $H_0: \lambda = \dfrac{1}{2}$ against $H_1: \lambda = \dfrac{3}{4}$ at the level $\alpha$ $(0 < \alpha < 1)$, which of the following statements is (are) TRUE?

(A) The most powerful test exists for each value of $\alpha$

(B) The most powerful test does not exist for some values of $\alpha$

(C) If the most powerful test exists, it is of the form: Reject $H_0$ if $X_1^2 + X_2^2 + \cdots + X_n^2 \leq c$ for some $c > 0$

(D) If the most powerful test exists, it is of the form: Reject $H_0$ if $X_1^2 + X_2^2 + \cdots + X_n^2 \geq c$ for some $c > 0$

## SECTION – C
## NUMERICAL ANSWER TYPE (NAT)

**Q. 41 – Q. 50 carry one mark each.**

Q.41 $\displaystyle\lim_{n\to\infty}\frac{\sqrt{n+1}+\sqrt{n+2}+\cdots+\sqrt{n+n}}{n\sqrt{n}}$ (round off to 2 decimal places) equals _____

Q.42 Let $f:[0,2]\to\mathbb{R}$ be such that $|f(x)-f(y)|\le|x-y|^{4/3}$ for all $x,y\in[0,2]$.
If $\int_0^2 f(x)dx=\frac{2}{3}$, then $\sum_{k=1}^{2019} f\left(\frac{1}{k}\right)$ equals _____

Q.43 The value (round off to 2 decimal places) of the double integral
$$\int_0^9 \int_{\sqrt{x}}^3 \frac{1}{1+y^3}\,dy\,dx$$
equals _____

Q.44 If $\begin{bmatrix}\dfrac{\sqrt{5}}{3} & -\dfrac{2}{3} & c \\ \dfrac{2}{3} & \dfrac{\sqrt{5}}{3} & d \\ a & b & 1\end{bmatrix}$ is a real orthogonal matrix, then $a^2+b^2+c^2+d^2$ equals _____

Q.45 Two fair dice are tossed independently and it is found that one face is odd and the other one is even. Then the probability (round off to 2 decimal places) that the sum is less than 6 equals _____

Q.46 Let $X$ be a random variable with the moment generating function
$$M_X(t)=\left(\frac{e^{\frac{t}{2}}+e^{-\frac{t}{2}}}{2}\right)^2, \qquad -\infty<t<\infty.$$

Using Chebyshev's inequality, the upper bound for $P\left(|X|>\sqrt{\dfrac{2}{3}}\right)$ equals _____

Q.47   In a production line of a factory, each packet contains four items. Past record shows that 20% of the produced items are defective. A quality manager inspects each item in a packet and approves the packet for shipment if at most one item in the packet is found to be defective. Then the probability (round off to 2 decimal places) that out of the three randomly inspected packets at least two are approved for shipment equals _____

Q.48   Let $X$ be the number of heads obtained in a sequence of 10 independent tosses of a fair coin. The fair coin is tossed again $X$ number of times independently, and let $Y$ be the number of heads obtained in these $X$ number of tosses. Then $E(X + 2Y)$ equals _____

Q.49   Let $0, 1, 0, 0, 1$ be the observed values of a random sample of size five from a discrete distribution with the probability mass function $P(X = 1) = 1 - P(X = 0) = 1 - e^{-\lambda}$, where $\lambda > 0$. The method of moments estimate (round off to 2 decimal places) of $\lambda$ equals _____

Q.50   Let $X_1, X_2, X_3$ be a random sample from $N(\mu_1, \sigma^2)$ distribution and $Y_1, Y_2, Y_3$ be a random sample from $N(\mu_2, \sigma^2)$ distribution. Also, assume that $(X_1, X_2, X_3)$ and $(Y_1, Y_2, Y_3)$ are independent. Let the observed values of $\sum_{i=1}^{3}\left[X_i - \frac{1}{3}(X_1 + X_2 + X_3)\right]^2$ and $\sum_{i=1}^{3}\left[Y_i - \frac{1}{3}(Y_1 + Y_2 + Y_3)\right]^2$ be 1 and 5, respectively. The length (round off to 2 decimal places) of the shortest 90% confidence interval of $\mu_1 - \mu_2$ equals _____

## Q. 51 – Q. 60 carry two marks each.

Q.51   $\lim_{n\to\infty}\left[n - \frac{n}{e}\left(1 + \frac{1}{n}\right)^n\right]$ equals _____

Q.52   For any real number $y$, let $[y]$ be the greatest integer less than or equal to $y$ and let $\{y\} = y - [y]$. For $n = 1, 2, ...,$ and for $x \in \mathbb{R}$, let

$$f_{2n}(x) = \begin{cases} \left[\frac{\sin x}{x}\right], & x \neq 0, \\ 1, & x = 0, \end{cases} \quad \text{and} \quad f_{2n-1}(x) = \begin{cases} \left\{\frac{\sin x}{x}\right\}, & x \neq 0, \\ 1, & x = 0. \end{cases}$$

Then $\lim_{x\to 0}\sum_{k=1}^{100} f_k(x)$ equals _____

Q.53   The volume (round off to 2 decimal places) of the region in the first octant $(x \geq 0, y \geq 0, z \geq 0)$ bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 2$ and $y + z = 4$ equals _____

Q.54

If $ad - bc = 2$ and $ps - qr = 1$, then the determinant of $\begin{bmatrix} a & b & 0 & 0 \\ 3 & 10 & 2p & q \\ c & d & 0 & 0 \\ 2 & 7 & 2r & s \end{bmatrix}$

equals _____

Q.55   In an ethnic group, 30% of the adult male population is known to have heart disease. A test indicates high cholesterol level in 80% of adult males with heart disease. But the test also indicates high cholesterol levels in 10% of the adult males with no heart disease. Then the probability (round off to 2 decimal places), that a randomly selected adult male from this population does not have heart disease given that the test indicates high cholesterol level, equals _____

Q.56   Let $X$ be a continuous random variable with the probability density function
$$f(x) = \begin{cases} ax^2, & 0 < x < 1, \\ bx^{-4}, & x \geq 1, \\ 0, & \text{otherwise,} \end{cases}$$
where $a$ and $b$ are positive real numbers. If $E(X) = 1$, then $E(X^2)$ equals _____

Q.57   Let $X$ and $Y$ be jointly distributed continuous random variables, where $Y$ is positive valued with $E(Y^2) = 6$. If the conditional distribution of $X$ given $Y = y$ is $U(1 - y, 1 + y)$, then $Var(X)$ equals _____

Q.58   Let $X_1, X_2, \ldots, X_{10}$ be i.i.d. $N(0, 1)$ random variables. If $T = X_1^2 + X_2^2 + \cdots + X_{10}^2$, then $E\left(\dfrac{1}{T}\right)$ equals _____

Q.59   Let $X_1, X_2, X_3$ be a random sample from a continuous distribution with the probability density function
$$f(x) = \begin{cases} e^{-(x-\mu)}, & x > \mu, \\ 0, & \text{otherwise.} \end{cases}$$
Let $X_{(1)} = \min\{X_1, X_2, X_3\}$ and $c > 0$ be a real number. Then $\left(X_{(1)} - c, X_{(1)}\right)$ is a 97% confidence interval for $\mu$, if $c$ (round off to 2-decimal places) equals _____

Q.60 Let $X_1, X_2, X_3, X_4$ be a random sample from a discrete distribution with the probability mass function $P(X = 0) = 1 - P(X = 1) = 1 - p$, for $0 < p < 1$. To test the hypothesis
$$H_0: p = \frac{3}{4} \quad \text{against} \quad H_1: p = \frac{4}{5},$$
consider the test:
$$\text{Reject } H_0 \text{ if } X_1 + X_2 + X_3 + X_4 > 3.$$
Let the size and power of the test be denoted by $\alpha$ and $\gamma$, respectively. Then $\alpha + \gamma$ (round off to 2 decimal places) equals _____

# END OF THE QUESTION PAPER